

# Class XII Session 2025-26

## Subject - Mathematics

### Sample Question Paper - 9

Time Allowed: 3 hours

Maximum Marks: 80

#### General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

#### Section A

1.  $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ , then the value of x is [1]  
a) 0 b) 7  
c) 3 d) 10
2. If  $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ , then  $\det(\text{adj } A)$  equals [1]  
a)  $a^{27}$  b)  $a^2$   
c)  $a^6$  d)  $a^9$
3. The system of equations,  $x + y + z = 1$ ,  $3x + 6y + z = 8$ ,  $\alpha x + 2y + 3z = 1$  has a unique solution for [1]  
a)  $\alpha$  not equal to 0 b) all rational  $\alpha$   
c) all real  $\alpha$  d) all integral  $\alpha$
4. If  $f(x) = x^n$ , then the value of  $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} \dots + \frac{(-1)^n f^n(1)}{n!}$  is [1]  
a) 1 b) 0  
c)  $2^n$  d) 2

[1]

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- a)  $xe^{\int P_1 dy} = \int (Q_1 e^{\int P_1 dy}) dy + C$       b)  $ye^{\int P_1 dy} = \int (Q_1 e^{\int P_1 dy}) dy + C$   
 c)  $xe^{\int P_1 dx} = \int (Q_1 e^{\int P_1 dx}) dx + C$       d)  $y \cdot e^{\int P_1 dx} = \int (Q_1 e^{\int P_1 dx}) dx + C$
16. If  $\vec{a} \cdot \hat{i} = \vec{a} \cdot (\hat{i} + \hat{j}) = \vec{a} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ , then  $\vec{a}$  is [1]  
 a)  $\hat{k}$       b)  $\hat{j}$   
 c)  $\hat{i} + \hat{j} + \hat{k}$       d)  $\hat{i}$
17. For what value of k may the function  $f(x) = \begin{cases} k(3x^2 - 5x), & x \leq 0 \\ \cos x, & x > 0 \end{cases}$  become continuous? [1]  
 a) 1      b)  $-\frac{1}{2}$   
 c) 0      d) No value
18. If a line makes an angle of  $30^\circ$  with the positive direction of x-axis,  $120^\circ$  with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is: [1]  
 a)  $0^\circ$       b)  $120^\circ$   
 c)  $90^\circ$       d)  $60^\circ$
19. **Assertion (A):** The average rate of change of the function  $y = 15 - x^2$  between  $x = 2$  and  $x = 3$  is -5. [1]  
**Reason (R):** Average rate of change  $\delta_y = y_{\text{at } x=3} - y_{\text{at } x=2}$ .  
 a) Both A and R are true and R is the correct explanation of A.      b) Both A and R are true but R is not the correct explanation of A.  
 c) A is true but R is false.      d) A is false but R is true.
20. **Assertion (A):** Let  $A = \{1, 5, 8, 9\}$ ,  $B = \{4, 6\}$  and  $f = \{(1, 4), (5, 6), (8, 4), (9, 6)\}$ , then f is a bijective function. [1]  
**Reason (R):** Let  $A = \{1, 5, 8, 9\}$ ,  $B = \{4, 6\}$  and  $f = \{(1, 4), (5, 6), (8, 4), (9, 6)\}$ , then f is a surjective function.  
 a) Both A and R are true and R is the correct explanation of A.      b) Both A and R are true but R is not the correct explanation of A.  
 c) A is true but R is false.      d) A is false but R is true.

### Section B

21. Write the interval for the principal value of function and draw its graph:  $\cot^{-1} x$ . [2]  
 OR  
 Find the value of  $\tan^{-1} \left( -\frac{1}{\sqrt{3}} \right) + \cot^{-1} \left( \frac{1}{\sqrt{3}} \right) + \tan^{-1} \left[ \sin \left( \frac{-\pi}{2} \right) \right]$ .
22. Show that  $f(x) = x^3 - 15x^2 + 75x - 50$  is increasing on R. [2]  
 23. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm. [2]  
 OR  
 Is the function  $\cos 3x$  decreasing on  $(0, \frac{\pi}{2})$ ?
24. Evaluate:  $\int x^2 \cos x dx$ . [2]  
 25. Find the maximum and minimum value,  $f(x) = |x + 2| - 1$  [2]

### Section C

26. Find  $\int \frac{5x-2}{1+2x+3x^2} dx$ . [3]  
 27. Probabilities of solving a specific problem independently by A and Bare  $\frac{1}{2}$  and  $\frac{1}{3}$  respectively. If both try to [3]

solve problem independently, then find the probability that

i. problem is solved.

ii. exactly one of them solves the problem.

28. If  $f(2a - x) = -f(x)$ , prove that  $\int_0^{2a} f(x) dx = 0$  [3]

OR

Evaluate:  $\int e^{\sin x} \sin 2x dx$

29. Find the particular solution of the differential equation  $x \frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$ , given that  $y(1) = 0$ . [3]

OR

Find the general solution of the differential equation:  $(1 - x^2) \frac{dy}{dx} + xy = ax$

30. Solve the Linear Programming Problem graphically: [3]

Minimize  $Z = 3x_1 + 5x_2$

Subject to

$$x_1 + 3x_2 \geq 3$$

$$x_1 + x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

OR

Solve the Linear Programming Problem graphically:

Minimize  $Z = 18x + 10y$

Subject to

$$4x + y > 20$$

$$2x + 3y > 30$$

$$x, y > 0$$

31. Find all points of discontinuity of  $f$  where  $f$  is defined as follows,  $f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$  [3]

#### Section D

32. Find the area of the smaller region bounded by the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and the line  $\frac{x}{3} + \frac{y}{2} = 1$ . [5]

33. If  $A = \{1, 2, 3, 4\}$ , define relations on  $A$  which have properties of being: [5]

a. reflexive, transitive but not symmetric

b. symmetric but neither reflexive nor transitive

c. reflexive, symmetric and transitive.

OR

Let  $A = [-1, 1]$ . Then, discuss whether the following functions defined on  $A$  are one-one, onto or bijective:

i.  $f(x) = \frac{x}{2}$

ii.  $g(x) = |x|$

iii.  $h(x) = x|x|$

iv.  $k(x) = x^2$

34. Solve the system of the following equations: (Using matrices): [5]

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4; \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1; \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2;$$

35. Find the vector equation of the line passing through  $(1, 2, 3)$  and parallel to each of the planes [5]

$$\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5 \text{ and } \vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6. \text{ Also find the point of intersection of the line thus obtained}$$

with the plane  $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$ .

OR

Find the shortest distance between the given lines.  $\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$ ,  $\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$

### Section E

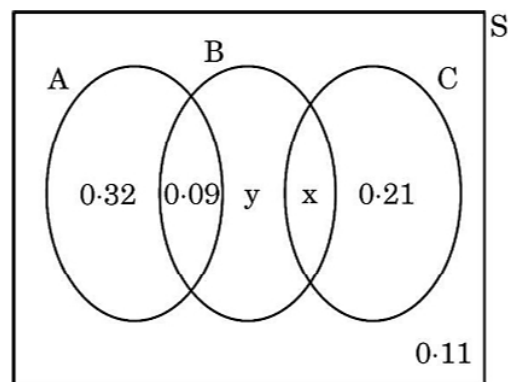
36. Read the following text carefully and answer the questions that follow:

[4]

There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below:



The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44.



- Find the value of x. (1)
- Find the value of y. (1)
- Find  $P\left(\frac{C}{B}\right)$ . (2)

OR

Find the probability that a randomly selected person of the society does Yoga of type A or B but not C. (2)

37. Read the following text carefully and answer the questions that follow:

[4]

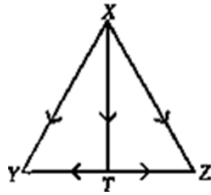
If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the

third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

- i. If  $\vec{p}, \vec{q}, \vec{r}$  are the vectors represented by the sides of a triangle taken in order, then find  $\vec{q} + \vec{r}$ . (1)
- ii. If ABCD is a parallelogram and AC and BD are its diagonals, then find the value of  $\vec{AC} + \vec{BD}$ . (1)
- iii. If ABCD is a parallelogram, where  $\vec{AB} = 2\vec{a}$  and  $\vec{BC} = 2\vec{b}$ , then find the value of  $\vec{AC} - \vec{BD}$ . (2)

**OR**

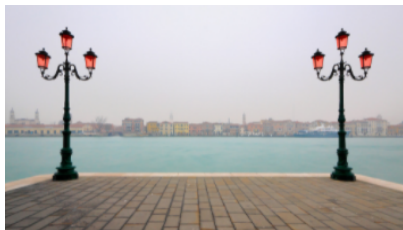
If T is the mid point of side YZ of  $\triangle XYZ$ , then what is the value of  $\vec{XY} + \vec{XZ}$ . (2)



38. **Read the following text carefully and answer the questions that follow:**

**[4]**

In a street two lamp posts are 600 feet apart. The light intensity at a distance  $d$  from the first (stronger) lamp post is  $\frac{1000}{d^2}$ , the light intensity at distance  $d$  from the second (weaker) lamp post is  $\frac{125}{d^2}$  (in both cases the light intensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.



- i. If  $I(x)$  denotes the combined light intensity, then find the value of  $x$  so that  $I(x)$  is minimum. (1)
- ii. Find the darkest spot between the two lights. (1)
- iii. If you are in between the lamp posts, at distance  $x$  feet from the stronger light, then write the combined light intensity coming from both lamp posts as function of  $x$ . (2)

**OR**

Find the minimum combined light intensity? (2)

# Solution

## Section A

1.

(c) 3

**Explanation:**

$$\text{Given, } x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\Rightarrow 2x - y = 10 \dots(i)$$

$$\text{and } 3x + y = 5 \dots(ii)$$

On solving Eqs. (i) and (ii), we get  $x = 3$

2.

(c)  $a^6$

**Explanation:**

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

$$|A| = a^3$$

$$|\text{adj } A| = |A|^{3-1} = |A|$$

$$|\text{adj } A| = (a^3)^2 = a^6$$

3.

(d) all integral  $\alpha$

**Explanation:**

The given system of equations has unique solution, if:

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 6 & 1 \\ \alpha & 2 & 3 \end{vmatrix} \neq 0$$

$$\Rightarrow 1(18 - 2) - 1(9 - \alpha) + 1(6 - 6\alpha) \neq 0$$

$$\Rightarrow 13 - 5\alpha \neq 0$$

$$\Rightarrow \alpha \neq \frac{13}{5}. \text{ (Since } \alpha \text{ is not integral value)}$$

Thus, unique solution exists for all integral values of  $\alpha$ .

4.

(b) 0

**Explanation:**

0

5. (a) 4, 5, 7

**Explanation:**

We have,

$$\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$$

$$\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-4}{-2}$$

The direction ratios of the given lines are proportional to 2, -3, 1 and 1, 2, -2.

The vectors parallel to the given lines are  $\vec{b}_1 = 2\hat{i} - 3\hat{j} + \hat{k}$  and  $\vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$

Vector perpendicular to the vectors  $b_1$  &  $b_2$  is ,

$$\begin{aligned}\vec{b} &= \vec{b}_1 \times \vec{b}_2 \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 2 & -2 \end{vmatrix} \\ &= 4\hat{i} + 5\hat{j} + 7\hat{k}\end{aligned}$$

Hence, the direction ratios of the line perpendicular to the given two lines are proportional to 4, 5, 7.

6.

(c)  $x^2 + y^2 = Cx$

**Explanation:**

$$(x^2 - y^2)dx + 2xydy = 0 \Rightarrow \frac{dy}{dx} = \frac{y^3 - x^3}{2xy}$$

Put  $y = vx$ , we have;  $\frac{dy}{dx} = v + x \frac{dv}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \Rightarrow v + x \frac{dv}{dx} = \frac{v^2 x^2 - x^2}{2xvx} \Rightarrow x \frac{dv}{dx} = - \left( \frac{v^2 + 1}{2v} \right)$$

$$\Rightarrow \int \frac{2v}{v^2 + 1} dv = - \int \frac{1}{x} dx \Rightarrow \log |v^2 + 1| = - \log |x| + \log C$$

$$\Rightarrow \log |v^2 + 1| + \log |x| = \log C \Rightarrow (v^2 + 1)|x| = C \Rightarrow (x^2 + y^2) = Cx$$

7.

(b) convex polygon

**Explanation:**

Feasible region for an LPP is always a convex polygon.

8.

(b)  $-3\hat{i} + 2\hat{j}$

**Explanation:**

Given that,  $\alpha = \hat{k}$

and  $\gamma = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Since,  $\beta$  is perpendicular to both  $\alpha$  and  $\gamma$ .

$$\begin{aligned}\text{i.e., } \beta &= \pm(\alpha \times \gamma) = \pm \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 2 & 3 & 4 \end{vmatrix} \\ &= \pm \hat{i}(0 - 3) - \hat{j}(0 - 2) + \hat{k}(0 - 0) \\ &= \pm(-3\hat{i} + 2\hat{j})\end{aligned}$$

9.

(a)  $\frac{\pi}{2}$

**Explanation:**

We have,

$$I = \int_0^1 \frac{d}{dx} \left\{ \sin^{-1} \left( \frac{2x}{1+x^2} \right) \right\} dx$$

We know since  $\int f'(x) = f(x)$

$$f(x) = \sin^{-1} \left( \frac{2x}{1+x^2} \right) \text{ and } f'(x) = \frac{d}{dx} \left\{ \sin^{-1} \left( \frac{2x}{1+x^2} \right) \right\}$$

$$\text{Therefore, } I = \left[ \sin^{-1} \left( \frac{2x}{1+x^2} \right) \right]_0^1$$

$$= \sin^{-1}(1) - \sin^{-1}(0)$$

$$= \frac{\pi}{2}$$

10.

(b)  $\frac{1}{3} \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

**Explanation:**



$$A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \dots(i)$$

$$A - 2B = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \dots(ii)$$

adding  $2 \times (i)$  and  $(ii)$ , we get

$$2A + 2B = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} \dots(iii)$$

$$A - 2B = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \dots(iv)$$

adding  $(iii)$  and  $(iv)$ , we get

$$\Rightarrow 3A = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\Rightarrow A = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$$

11.

(b) (2, 3)

**Explanation:**

Since (2, 3) does not satisfy  $2x + 3y - 12 \leq 0$  as  $2 \times 2 + 3 \times 3 - 12 = 4 + 9 - 12 = 1 \neq 0$

12.

(b)  $\alpha$

**Explanation:**

Let  $\alpha = x\hat{i} + y\hat{j} + z\hat{k}$

Then  $(\alpha \cdot \hat{i})\hat{i} + (\alpha \cdot \hat{j})\hat{j} + (\alpha \cdot \hat{k})\hat{k}$

$$= \{(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{i}\}\hat{i} + \{(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{j}\}\hat{j} + \{(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{k}\}\hat{k}$$

$$= (x)\hat{i} + (y)\hat{j} + (z)\hat{k} = \alpha$$

13. (a)  $a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$

**Explanation:**

By the definition of expansion of determinant, the required relation is

$$a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$$

14. (a)  $\frac{7}{12}$

**Explanation:**

Here,  $P(A) = \frac{3}{10}$ ,  $P(B) = \frac{2}{5}$  and  $P(A \cup B) = \frac{3}{5}$

$$P(B/A) + P(A/B) = \frac{P(B \cap A)}{P(A)} + \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A) + P(B) - P(A \cup B)}{P(A)} + \frac{P(A) + P(B) - P(A \cup B)}{P(B)}$$

$$= \frac{\frac{3}{10} + \frac{2}{5} - \frac{3}{5}}{\frac{3}{10}} + \frac{\frac{3}{10} + \frac{2}{5} - \frac{3}{5}}{\frac{2}{5}}$$

$$= \frac{\frac{1}{10}}{\frac{3}{10}} + \frac{\frac{1}{10}}{\frac{2}{5}} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

15. (a)  $x e^{\int P_1 dy} = \int (Q_1 e^{\int P_1 dy}) dy + C$

**Explanation:**

The integrating factor of the given differential equation

$$\frac{dx}{dy} + P_1 x = Q_1 \text{ is } e^{\int P_1 dy}$$

Thus, the general solution of the differential equation is given by,

$$x(I.F.) = \int (Q_1 \times I.F.) dy + C$$

$$\Rightarrow x \cdot e^{\int P_1 dy} = \int (Q_1 e^{\int P_1 dy}) dy + C$$

16.

(d)  $\hat{i}$

**Explanation:**

$\hat{i}$

17.

(d) No value

**Explanation:**

No value

18.

(c)  $90^\circ$

**Explanation:**

To find the angle with the z-axis, we use the relation:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

where  $\alpha = 30^\circ$  and  $\beta = 120^\circ$ . Calculating, we get:

$$\cos^2 30^\circ = \frac{3}{4}, \cos^2 120^\circ = \frac{1}{4}$$

Thus,

$$\frac{3}{4} + \frac{1}{4} + \cos^2 \gamma = 1 \implies \cos^2 \gamma = 0 \implies \gamma = 90^\circ.$$

So, the angle with the z-axis is  $90^\circ$ .

19. (a) Both A and R are true and R is the correct explanation of A.

**Explanation:**

$$\text{Let } y = f(x) = 15 - x^2$$

If x changes from 2 to 3 then  $\delta_x = 3 - 2 = 1$

$$\text{Again } f(3) = 15 - 9 = 6 \text{ and } f(2) = 15 - 4 = 11$$

$$\text{Therefore } \delta_y = f(3) - f(2) = 6 - 11 = -5$$

20.

(d) A is false but R is true.

**Explanation:**

We have,  $A = \{1, 5, 8, 9\}$ ,  $B = \{4, 6\}$  and  $f = \{(1, 4), (5, 6), (8, 4), (9, 6)\}$

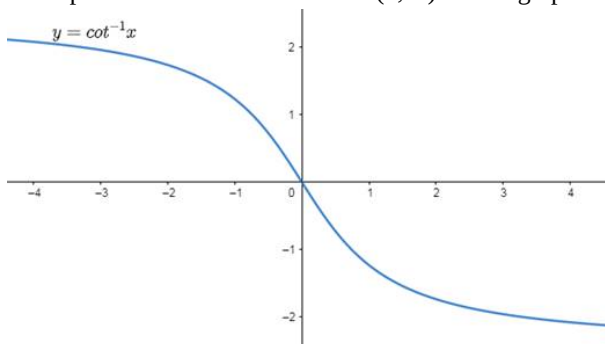
So, all elements of B has a domain element on A or we can say elements 1 and 8 & 5 and 9 have some range 4 & 6, respectively.

Therefore,  $f : A \rightarrow B$  is a surjective function not one to one function.

Also, for a bijective function, f must be both one to one onto.

### Section B

21. Principal value branch of  $\cot^{-1} x$  is  $(0, \pi)$  and its graph is shown below.



OR

$$\begin{aligned} \text{We have, } & \tan^{-1} \left( -\frac{1}{\sqrt{3}} \right) + \cot^{-1} \left( \frac{1}{\sqrt{3}} \right) + \tan^{-1} \left[ \sin \left( -\frac{\pi}{2} \right) \right] \\ &= \tan^{-1} \left( \tan \frac{5\pi}{6} \right) + \cot^{-1} \left( \cot \frac{\pi}{3} \right) + \tan^{-1} (-1). \\ &= \tan^{-1} \left[ \tan \left( \pi - \frac{\pi}{6} \right) \right] + \cot^{-1} \left[ \cot \left( \frac{\pi}{3} \right) \right] + \tan^{-1} \left[ \tan \left( \pi - \frac{\pi}{4} \right) \right] \end{aligned}$$

$$= \tan^{-1}\left(-\tan \frac{\pi}{6}\right) + \cot^{-1}\left(\cot \frac{\pi}{3}\right) + \tan^{-1}\left(-\tan \frac{\pi}{4}\right) \left[ \begin{array}{l} \because \tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \cot^{-1}(\cot x) = x, x \in (0, \pi) \\ \text{and } \tan^{-1}(-x) = -\tan^{-1}x \end{array} \right]$$

$$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{-2\pi + 4\pi - 3\pi}{12}$$

$$= \frac{-5\pi + 4\pi}{12} = \frac{-\pi}{12}$$

22. Consider the given function,

$$f(x) = x^3 - 15x^2 + 75x - 50$$

Domain of the function is R.

$$f'(x) = 3x^2 - 30x + 75$$

$$= 3(x^2 - 10x + 25)$$

$$= 3(x - 5)(x - 5)$$

$$= 3(x - 5)^2$$

$$f'(x) = 0 \text{ for } x = 5$$

$$\text{for } x < 5$$

$$f'(x) > 0$$

and

$$\text{for } x > 5$$

$$f'(x) > 0$$

we can see throughout R the derivative is +ve but at  $x = 5$  it is 0 so it is increasing.

23. Let  $r$  be the radius of sphere and  $V$  be its volume.

$$\text{Then } V = \frac{4}{3}\pi r^3 \dots\dots(i)$$

$$\text{Given, } \frac{dV}{dt} = 3 \text{ cm}^3/\text{s}$$

Differentiating (i) both sides w.r.t  $x$ , we get,

$$\frac{dV}{dt} = \frac{4}{3}\pi (3r^2) \frac{dr}{dt}$$

$$\Rightarrow 3 = \frac{4}{3}(3\pi r^2) \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{3}{4\pi r^2} \dots\dots(ii)$$

Now, let  $S$  be the surface area of sphere, then

$$S = 4\pi r^2$$

$$\Rightarrow \frac{dS}{dt} = 4\pi (2r) \frac{dr}{dt}$$

$$\Rightarrow \frac{dS}{dt} = 8\pi r \left( \frac{3}{4\pi r^2} \right) [\text{using Eq.(ii)}]$$

$$\Rightarrow \left( \frac{dS}{dt} \right) = \frac{6}{r}$$

when  $r = 2$ , then

$$\frac{dS}{dt} = \frac{6}{2} = 3 \text{ cm}^2/\text{s}$$

Therefore, the rate of increase of the surface area of sphere is  $3 \text{ cm}^2/\text{s}$  when its radius is 2 cm.

OR

$$\text{Let } f(x) = \cos 3x$$

$$\therefore f'(x) = -3 \sin 3x$$

$$\text{Now, } f'(x) = 0$$

$$\Rightarrow \sin 3x = 0$$

$$\Rightarrow 3x = \pi, \text{ as } x \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow x = \frac{\pi}{3}$$

The point  $x = \frac{\pi}{3}$  divides the interval  $\left(0, \frac{\pi}{2}\right)$  into two distinct intervals.

$$\text{i.e. } \left(0, \frac{\pi}{3}\right) \text{ and } \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$$

$$\text{Now, in the interval, } \left(0, \frac{\pi}{3}\right)$$

$$f'(x) = -3 \sin 3x < 0 \text{ as } \left(0 < x < \frac{\pi}{3} \Rightarrow 0 < 3x < \pi\right)$$

$$\text{Therefore, 'f' is strictly decreasing in interval } \left(0, \frac{\pi}{3}\right)$$

$$\text{Now, in the interval } \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$$

$$f'(x) = -3 \sin 3x > 0 \text{ as } \frac{\pi}{3} < x < \frac{\pi}{2} \Rightarrow \pi < 3x < \frac{3\pi}{2}$$

Therefore, 'f' is strictly increasing in the interval  $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ .

24. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here  $x^2$  is the first function, and  $\cos x$  is the second function.

Using Integration by part

$$\int a \cdot b \cdot dx = a \int b dx - \int \left[ \frac{da}{dx} \cdot \int b dx \right] dx$$

$$\Rightarrow \int x^2 \cos x dx = x^2 \int \cos x dx - \int \left[ \frac{dx^2}{dx} \cdot \int \cos x dx \right] dx$$

$$= x^2 \sin x - \int [2x \times \sin x] dx$$

$$= x^2 \sin x - 2 \left[ \int x \sin x dx \right]$$

Again applying by the part method in the second half, we get

$$x^2 \sin x - 2 \int x \sin x dx$$

$$= x^2 \sin x - 2 \left[ x \int \sin x dx - \int \left( \frac{dx}{dx} \cdot \int \sin x dx \right) dx \right]$$

$$= x^2 \sin x - 2 [x(-\cos x) - \int 1 \cdot (-\cos x) dx]$$

$$= x^2 \sin x - 2[-x \cos x + \sin x] + c$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x + c$$

25. It is given that  $f(x) = |x + 2| - 1$

Now, we can see that  $|x + 2| \geq 0$  for every  $x \in \mathbb{R}$

$$\Rightarrow f(x) = |x + 2| - 1 \geq -1 \text{ for every } x \in \mathbb{R}$$

Clearly, the minimum value of  $f$  is attained when  $|x + 2| = 0$

$$\text{i.e., } |x + 2| = 0$$

$$\Rightarrow x = -2$$

$$\text{Then, Minimum value of } f = f(-2) = |-2 + 2| - 1 = -1$$

Therefore, function  $f$  does not have a maximum value.

### Section C

26. According to the question,  $I = \int \frac{5x-2}{1+2x+3x^2} dx$

$(5x - 2)$  can be written as ,

$$5x - 2 = A \frac{d}{dx} (1 + 2x + 3x^2) + B$$

$$I = \int \frac{5x-2}{1+2x+3x^2} dx = \int \frac{A \frac{d}{dx} (1+2x+3x^2) + B}{1+2x+3x^2} dx \dots (i)$$

$$\Rightarrow 5x - 2 = A(2 + 6x) + B$$

Comparing the coefficients of  $x$  and constant terms,

$$5 = 6A \Rightarrow A = \frac{5}{6}$$

$$\text{and } -2 = 2A + B \Rightarrow B = -2A - 2$$

$$= -\frac{5}{3} - 2 = -\frac{11}{3} \left[ \because A = \frac{5}{6} \right]$$

From Eq. (i), we get

$$I = \int \frac{\frac{5}{6}(2+6x) - \frac{11}{3}}{1+2x+3x^2} dx$$

$$\Rightarrow I = \int \frac{\frac{5}{6}(2+6x)}{1+2x+3x^2} dx - \int \frac{\left(\frac{11}{3}\right)}{1+2x+3x^2} dx$$

$$\Rightarrow I = I_1 - I_2 \dots (ii)$$

$$\text{where, } I_1 = \frac{5}{6} \int \frac{2+6x}{1+2x+3x^2} dx$$

$$\text{Put } 1 + 2x + 3x^2 = t \Rightarrow (2 + 6x)dx = dt$$

$$\therefore I_1 = \frac{5}{6} \int \frac{dt}{t} = \frac{5}{6} \log |t| + C_1$$

$$= \frac{5}{6} \log |1 + 2x + 3x^2| + C_1 \quad [t = 1 + 2x + 3x^2]$$

$$\text{where, } I_2 = \frac{11}{3} \int \frac{dx}{3x^2 + 2x + 1}$$

$$= \frac{11}{9} \int \frac{dx}{\left[x^2 + \frac{2x}{3} + \frac{1}{3}\right]}$$

$$= \frac{11}{9} \int \frac{dx}{\left[x^2 + \frac{2x}{3} + \frac{1}{3} + \frac{1}{9} - \frac{1}{9}\right]}$$

$$\begin{aligned}
&= \frac{11}{9} \int \frac{dx}{\left[x^2 + \frac{2x}{3} + \frac{1}{9} + \frac{1}{3} - \frac{1}{9}\right]} \\
&= \frac{11}{9} \int \frac{dx}{\left(x + \frac{1}{3}\right)^2 + \frac{2}{9}} \left[\because (a+b)^2 = a^2 + b^2 + 2ab\right] \\
&= \frac{11}{9} \cdot \frac{1}{\frac{\sqrt{2}}{3}} \tan^{-1} \left( \frac{x + \frac{1}{3}}{\frac{\sqrt{2}}{3}} \right) + C_2 \left[ \because \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + c \right] \\
&= \frac{11}{3\sqrt{2}} \tan^{-1} \left( \frac{3x+1}{\sqrt{2}} \right) + C_2
\end{aligned}$$

Putting the values of  $I_1$  and  $I_2$  in Equation (ii),

$$\begin{aligned}
\Rightarrow I &= \frac{5}{6} \log|1 + 2x + 3x^2| + C_1 - \frac{11}{3\sqrt{2}} \tan^{-1} \left( \frac{3x+1}{\sqrt{2}} \right) - C_2 \\
\Rightarrow I &= \frac{5}{6} \log|1 + 2x + 3x^2| - \frac{11}{3\sqrt{2}} \tan^{-1} \left( \frac{3x+1}{\sqrt{2}} \right) + C \quad [\because C = C_1 - C_2]
\end{aligned}$$

27. Let  $P(A)$  = Probability that A solves the problem

$P(B)$  = Probability that B solves the problem

$P(\bar{A})$  = Probability that A does not solve the problem

and  $P(\bar{B})$  = Probability that B does not solve the problem

According to the question, we have

$$P(A) = \frac{1}{2}$$

$$\text{then } P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$[\because P(A) + P(\bar{A}) = 1]$$

$$\text{and } P(B) = \frac{1}{3}$$

$$\text{then } P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$$

i. P (problem is solved)

$$\begin{aligned}
&= P(A \cap \bar{B}) + P(\bar{A} \cap B) + P(A \cap B) \\
&= P(A)P(\bar{B}) + P(\bar{A}) \cdot P(B) + P(A) \cdot P(B)
\end{aligned}$$

$[\because A \text{ and } B \text{ are independent events}]$

$$\begin{aligned}
&= \left( \frac{1}{2} \times \frac{2}{3} \right) + \left( \frac{1}{2} \times \frac{1}{3} \right) + \left( \frac{1}{2} \times \frac{1}{3} \right) \\
&= \frac{2}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}
\end{aligned}$$

Hence, probability that the problem is solved, is  $\frac{2}{3}$

ii. P (exactly one of them solve the problem)

$$\begin{aligned}
&= P(A \text{ solve but } B \text{ do not solve}) + P(A \text{ do not solve but } B \text{ solve}) \\
&= P(A \cap \bar{B}) + P(\bar{A} \cap B) \\
&= P(A) \cdot P(\bar{B}) + P(\bar{A})P(B) \\
&= \left( \frac{1}{2} \times \frac{2}{3} \right) + \left( \frac{1}{2} \times \frac{1}{3} \right) = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}
\end{aligned}$$

28. We have, to write the above integral as,

$$I = \int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_a^{2a} f(x) dx$$

$$I = \int_0^a f(x) dx + I_1$$

For  $I_1$ , Let  $2a - t = x$  then  $dx = -dt$

$$t = a, x = a$$

$$t = 2a, x = 0$$

$$I_1 = \int_0^{2a} f(x) dx = \int_a^0 f(2a - t)(-dt)$$

$$= - \int_a^0 f(2a - t) dt$$

$$I_1 = \int_0^a f(2a - t) dt = \int_0^a f(2a - x) dx$$

$$I = \int_0^a f(x) dx + \int_0^a f(2a - x) dx$$

$$I = \int_0^a f(x) dx - \int_0^a f(x) dx \quad [\because f(2a - x) = -f(x)]$$

$$I = 0$$

Hence,

$$\int_0^{2a} f(x) dx = 0$$

OR

We can write  $\sin 2x = 2 \sin x \cos x$

$$\int e^{\sin x} \sin 2x dx = 2 \int e^{\sin x} \cdot \sin x \cos x dx$$

Let  $\sin x = t$

$\cos x dx = dt$

$$2 \int e^{\sin x} \sin x \cos x dx = 2 \int e^t t dt$$

Using BY PARTS METHOD.

$$2 \int e^t \cdot t dt = 2 \left[ t \int e^t dt - \int \left( \frac{dt}{dt} \cdot \int e^t dt \right) dt \right]$$

$$= 2 [t \cdot e^t - \int 1 \cdot e^t dt]$$

$$= 2 [t \cdot e^t - e^t] + c$$

$$= 2e^t(t - 1) + c$$

Replacing  $t$  with  $\sin x$

$$= 2e^{\sin x}(\sin x - 1) + c$$

$$29. x \frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$$

$$x \frac{dy}{dx} + y = -\frac{1}{1+x^2}$$

$$\frac{dy}{dx} + \left(\frac{1}{x}\right)y = -\frac{1}{x(1+x^2)}$$

So it is a linear differential equation

$$P = \frac{1}{x} \text{ and } Q = -\frac{1}{x(1+x^2)}$$

$$IF = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$$

General solution

$$y \cdot IF = \int Q \cdot IF dx + C$$

$$y \cdot x = -\int \frac{1}{x(1+x^2)} \cdot x dx + C$$

$$y \cdot x = -\int \frac{1}{(1+x^2)} dx + C$$

$$y \cdot x = -\tan^{-1} x + C \dots(i)$$

At  $x=1$ ,  $y=0$

$$0 = -\tan^{-1}(1) + C$$

$$C = \frac{\pi}{4}$$

Putting in eqn (i)

$$y \cdot x = -\tan^{-1} x + \frac{\pi}{4}$$

OR

The given differential equation is,

$$(1-x^2) \frac{dy}{dx} + xy = ax$$

$$\Rightarrow \frac{dy}{dx} + \frac{x}{1-x^2} \cdot y = \frac{ax}{1-x^2}$$

This is of the form  $\frac{dy}{dx} + Py = Q$

where  $P = \frac{x}{1-x^2}$  and  $Q = \frac{ax}{1-x^2}$

This given differential equation is linear

Now,  $IF = e^{\int P dx}$

$$= e^{\int \frac{x dx}{1-x^2}}$$

$$= e^{-\frac{1}{2} \int \frac{-2x}{1-x^2} dx} = e^{-\frac{1}{2} \log(1-x^2)} = (1-x^2)^{-\frac{1}{2}} = \frac{1}{\sqrt{1-x^2}}$$

Solution is

$$(IF) \cdot y = \int (IF)Q dx + C$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} \cdot y = \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{ax}{1-x^2} dx + C$$

$$\Rightarrow \frac{y}{\sqrt{1-x^2}} = \int \frac{ax}{(1-x^2)^{3/2}} dx + C$$

$$\Rightarrow \frac{y}{\sqrt{1-x^2}} = -\frac{a}{2} \int \frac{-2x}{(1-x^2)^{3/2}} dx + C \dots(i)$$

$$\text{Now, } I = \int \frac{-2x}{(1-x^2)^{3/2}} dx$$

Let  $t = 1 - x^2$

$$\Rightarrow dt = -2x dx$$

$$\therefore I = \int \frac{dt}{t^{3/2}} \Rightarrow I = \int t^{-3/2} dt$$

$$\Rightarrow I = \frac{\frac{-1}{t^2}}{\frac{-1}{2}} \Rightarrow I = -2 \cdot \frac{1}{\sqrt{t}} = \frac{-2}{\sqrt{1-x^2}}$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{-a}{2} \times \frac{-2}{\sqrt{1-x^2}} + C$$

$$\Rightarrow y = a + C\sqrt{1-x^2}$$

30. First, we will convert the given inequations into equations, we obtain the following equations:

$$x_1 + 3x_2 = 3, x_1 + x_2 = 2, x_1 = 0 \text{ and } x_2 = 0$$

Region represented by  $x_1 + 3x_2 \geq 3$  :

The line  $x_1 + 3x_2 = 3$  meets the coordinate axes at A(3,0) and B(0,1) respectively. By joining these points we obtain the line  $x_1 + 3x_2 = 3$

Clearly (0,0) does not satisfies the inequation  $x_1 + 3x_2 \geq 3$  . So, the region in the plane which does not contain the origin

represents the solution set of the inequation  $x_1 + 3x_2 \geq 3$  Region represented by  $x_1 + x_2 \geq 2$  The line  $x_1 + x_2 = 2$  meets the

coordinate axes at C(2,0) and D(0,2) respectively. By joining these points we obtain the

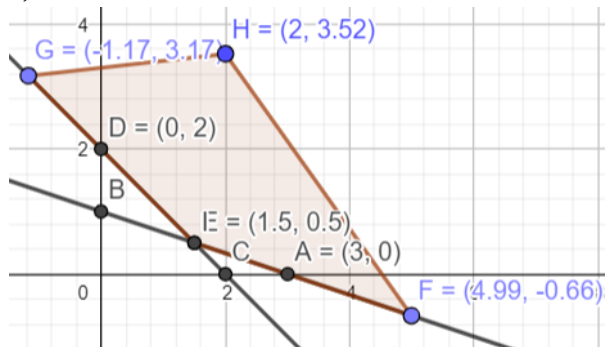
line  $x_1 + x_2 = 2$  Clearly (0,0) does not satisfies the inequation  $x_1 + x_2 \geq 2$  . So, the region containing the origin represents the

solution set of the inequation  $x_1 + x_2 \geq 2$  Region represented by  $x_1 \geq 0$  and  $x_2 \geq 0$  since, every point in the first quadrant

satisfies these inequations. So, the first quadrant is the region represented by the inequations  $x_1 \geq 0$  and  $x_2 \geq 0$  The feasible

region determined by subject to the constraints are,  $x_1 + 3x_2 \geq 3$ ,  $x_1 + x_2 \geq 2$ , and the non-negative restrictions  $x_1 \geq 0$ , and  $x_2 \geq$

0, are as follows



The corner points of the feasible region are O(0,0), B(0,1),  $E\left(\frac{3}{2}, \frac{1}{2}\right)$  and C(2,0)

The values of objective function at the corner points are as follows:

$$\text{Corner point : } z = 3x_1 + 5x_2$$

$$O(0, 0) : 3 \times 0 + 5 \times 0 = 0$$

$$B(0, 1) : 3 \times 0 + 5 \times 1 = 5$$

$$E\left(\frac{3}{2}, \frac{1}{2}\right) : \frac{3}{2} + 5 \times \frac{1}{2} = 7$$

$$C(2, 0) : 3 \times 2 + 5 \times 0 = 6$$

Therefore, the minimum value of objective function Z is 0 at the point O(0,0). Hence,  $x_1 = 0$  and  $x_2 = 0$  is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is 0.

OR

First, we will convert the given inequations into equations, we obtain the following equations:

$$4x + y = 20, 2x + 3y = 30, x = 0 \text{ and } y = 0$$

Region represented by  $4x + y \geq 20$  :

The line  $4x + y = 20$  meets the coordinate axes at A(5,0) and B(0,20) respectively. By joining these points we obtain the line  $4x + y = 20$

Clearly (0,0) does not satisfies the inequation  $4x + y \geq 20$  .

So, the region in xy plane which does not contain the origin represents the solution set of the inequation  $4x + y \geq 20$

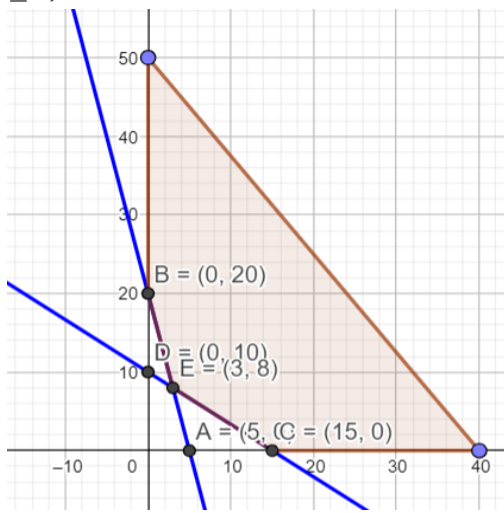
Region represented by  $2x + 3y \geq 30$  :

The line  $2x + 3y = 30$  meets the coordinate axes at C(15,0) and D(0,10) respectively. By joining these points we obtain the line  $2x + 3y = 30$  .

Clearly (0,0) does not satisfies the inequation  $2x + 3y \geq 30$  . So, the origin does not contain represents the solution set of the inequation  $2x + 3y \geq 30$ .

Region represented by  $x \geq 0$  and  $y \geq 0$  : graph will be in first quadrant

since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations  $x \geq 0$ , and  $y \geq 0$ . The feasible region determined by the system of constraints,  $4x + y \geq 20$ ,  $2x + 3y \geq 30$ ,  $x \geq 0$ , and  $y \geq 0$ , are as follows.



The corner points of the feasible region are B(0,20), C(15,0), E(3,8) and C(15,0)

The values of Z at these corner points are as follows.

The value of objective function at the corner point :  $Z = 18x + 10y$

$$B(0, 20) : 18 \times 0 + 10 \times 20 = 200$$

$$E(3, 8) : 18 \times 3 + 10 \times 8 = 134$$

$$C(15, 0) : 18 \times 15 + 10 \times 0 = 270$$

Therefore, the minimum value of Z is 134 at the point E(3,8) . Hence,  $x = 3$  and  $y = 8$  is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is 134.

31. Given function is

$$f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases} = \begin{cases} -x + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$$

First, we verify continuity at  $x = -3$  and then at  $x = 3$

**Continuity at  $x = -3$**

$$\text{LHL} = \lim_{x \rightarrow (-3)^-} f(x) = \lim_{x \rightarrow (-3)^-} (-x + 3)$$

$$\Rightarrow \text{LHL} = \lim_{h \rightarrow 0} [ -(-3 - h) + 3 ]$$

$$= \lim_{h \rightarrow 0} (3 + h + 3)$$

$$= 3 + 3 = 6$$

$$\text{and RHL} = \lim_{x \rightarrow (-3)^+} f(x) = \lim_{x \rightarrow (-3)^+} (-2x)$$

$$\Rightarrow \text{RHL} = \lim_{h \rightarrow 0} [ -2(-3 + h) ]$$

$$= \lim_{h \rightarrow 0} (6 - 2h)$$

$$\Rightarrow \text{RHL} = 6$$

Also,  $f(-3) = \text{value of } f(x) \text{ at } x = -3$

$$= -(-3) + 3$$

$$= 3 + 3 = 6$$

$$\therefore \text{LHL} = \text{RHL} = f(-3)$$

$\therefore f(x)$  is continuous at  $x = -3$  So,  $x = -3$  is the point of continuity.

Continuity at  $x = 3$

$$\text{LHL} = \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} [-(2x)]$$

$$\Rightarrow \text{LHL} = \lim_{h \rightarrow 0} [ -2(3 - h) ]$$

$$= \lim_{h \rightarrow 0} (-6 + 2h)$$

$$\Rightarrow \text{LHL} = -6$$

$$\text{and RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (6x + 2)$$



$$\Rightarrow \text{RHL} = \lim_{h \rightarrow 0} [6(3+h) + 2]$$

$$\Rightarrow \text{RHL} = 20$$

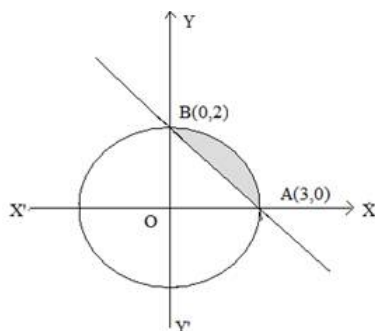
$$\therefore \text{LHL} \neq \text{RHL}$$

$\therefore f$  is discontinuous at  $x = 3$  Now, as  $f(x)$  is a polynomial function for  $x < -3$ ,  $-3 < x < 3$  and  $x > 3$  so it is continuous in these intervals.

Hence, only  $x = 3$  is the point of discontinuity of  $f(x)$ .

#### Section D

32.



$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \dots\dots\dots(1)$$

$$\frac{x}{3} + \frac{y}{2} = 1 \dots\dots\dots(2)$$

$\Rightarrow \frac{x^2}{(3)^2} + \frac{y^2}{(2)^2} = 1$  is the equation of ellipse and  $\frac{x}{3} + \frac{y}{2} = 1$  is the equation of intercept form of line.

On solving (1) and (2), we get points of intersection as (0,2) and (3,0).

$$\begin{aligned} \text{Area} &= \frac{2}{3} \int_0^3 \sqrt{9-x^2} dx - \int_0^3 \left( \frac{6-2x}{3} \right) dx \\ &= \frac{2}{3} \left[ \frac{x}{2} \sqrt{3^2-x^2} + \frac{3^2}{2} \sin^{-1} \frac{x}{3} \right]_0^3 - \frac{1}{3} \left[ 6x - \frac{2x^2}{2} \right]_0^3 \\ &= \frac{2}{3} \left[ \frac{3}{2} \sqrt{9-9} + \frac{9}{2} \sin^{-1} 1 \right] - \frac{1}{3} [18 - 9] \\ &= \frac{2}{3} \left( \frac{3}{2} \pi \right) - 3 \\ &= \frac{3}{2} (\pi - 2) \text{ sq unit.} \end{aligned}$$

33. Given that  $A = \{1, 2, 3, 4\}$ ,

a. Let  $R_1 = \{(1,1), (1,2), (2,3), (2,2), (1,3), (3,3)\}$

$R_1$  is reflexive, since, (1,1) (2,2) (3,3) lie in  $R_1$

Now,  $(1,2) \in R_1$   $(2,3) \in R_1 \Rightarrow (1,3) \in R_1$

Hence,  $R_1$  is also transitive but  $(1,2) \in R_1 \Rightarrow (2,1) \notin R_1$

So, it is not symmetric.

b. Let  $R_2 = \{(1,2), (2,1)\}$ . Here,  $1, 2, 3 \in \{1, 2, 3\}$  but  $(1,1), (2,2), (3,3)$  are not in  $R$ .

Therefore,  $R$  is not reflexive. Now,  $(1,2) \in R_2$ ,  $(2,1) \in R_2$

So, it is symmetric.

Now  $(1,2) \in R$   $(2,1) \in R$ , but  $(1,1) \notin R$ ,

therefore,  $R$  is not transitive.

c. Let  $R_3 = \{(1,2), (2,1), (1,1), (2,2), (3,3), (1,3), (3,1), (2,3)\}$

Clearly,  $R_3$  is reflexive, symmetric and transitive.

OR

Given that  $A = [-1, 1]$

i.  $f(x) = \frac{x}{2}$

Let  $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1}{2} = \frac{x_2}{2} \Rightarrow x_1 = x_2$$

So,  $f(x)$  is one-one.

Now, let  $y = \frac{x}{2}$

$$\Rightarrow x = 2y \notin A, \forall y \in A$$

As for  $y = 1 \in A$ ,  $x = 2 \notin A$

So,  $f(x)$  is not onto.

Also,  $f(x)$  is not bijective as it is not onto.

ii.  $g(x) = |x|$

Let  $g(x_1) = g(x_2)$

$\Rightarrow |x_1| = |x_2| \Rightarrow x_1 = \pm x_2$

So,  $g(x)$  is not one-one.

Now,  $x = \pm y \notin A$  for all  $y \in \mathbb{R}$

So,  $g(x)$  is not onto, also,  $g(x)$  is not bijective.

iii.  $h(x) = x|x|$

$\Rightarrow x_1|x_1| = x_2|x_2| \Rightarrow x_1 = x_2$

So,  $h(x)$  is one-one

Now, let  $y = x|x|$

$\Rightarrow y = x^2 \in A, \forall x \in A$

So,  $h(x)$  is onto also,  $h(x)$  is a bijective.

iv.  $k(x) = x^2$

Let  $k(x_1) = k(x_2)$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

Thus,  $k(x)$  is not one-one.

Now, let  $y = x^2$

$\Rightarrow x\sqrt{y} \notin A, \forall y \in A \quad x = \sqrt{y} \notin A, \forall y \in A$

As for  $y = -1, x = \sqrt{-1} \notin A$

Hence,  $k(x)$  is neither one-one nor onto.

34. Put  $\frac{1}{x} = u, \frac{1}{y} = v$  and  $\frac{1}{z} = w$  in the given equations,

$2u + 3v + 10w = 4; 4u - 6v + 5w = 1; 6u + 9v - 20w = 2$

$\therefore$  The matrix form of given equations is  $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} x \\ v \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} [AX = B]$

Here,  $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X \begin{bmatrix} x \\ v \\ z \end{bmatrix}$  and  $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$

$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$

$= 150 + 330 + 750 = 1200 \neq 0$

$\therefore A^{-1}$  exists and unique solution is  $X = A^{-1}B \dots (i)$

Now  $A_{11} = 75, A_{12} = 110, A_{13} = 72$  and  $A_{21} = 150, A_{22} = -100, A_{23} = 0$  and  $A_{31} = 75, A_{32} = 30, A_{33} = -24$

$\therefore \text{adj. } A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}' = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

And  $A^{-1} = \frac{\text{adj. } A}{|A|} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

$\therefore$  From eq. (i),

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$

$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$

$$= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\therefore u = \frac{1}{2}, v = \frac{1}{3}, w = \frac{1}{5}$$

$$\Rightarrow x = \frac{1}{u} = 2, y = \frac{1}{v} = 3, z = \frac{1}{w} = 5$$

35. Here the equation of two planes are:  $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$  and  $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$ .

Since the line is parallel to the two planes.

$$\therefore \text{Direction of line } \vec{b} = (\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + \hat{k})$$

$$= -3\hat{i} + 5\hat{j} + 4\hat{k}$$

$\therefore$  Equation of required line is

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k}) \dots\dots\dots (i)$$

Any point on line (i) is  $(1 - 3\lambda, 2 + 5\lambda, 3 + 4\lambda)$

For this line to intersect the plane  $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k})$  we have

$$(1 - 3\lambda)2 + (2 + 5\lambda)1 + (3 + 4\lambda)1 = 4$$

$$\Rightarrow \lambda = 1$$

$\therefore$  Point of intersection is  $(4, -3, -1)$

OR

Here, it is given that

$$\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$$

$$\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$$

Here, we have

$$\vec{a}_1 = 6\hat{i} + 3\hat{k}$$

$$\vec{b}_1 = 2\hat{i} - \hat{j} + 4\hat{k}$$

$$\vec{a}_2 = -9\hat{i} + \hat{j} - 10\hat{k}$$

$$\vec{b}_2 = 4\hat{i} + \hat{j} + 6\hat{k}$$

Thus,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ 4 & 1 & 6 \end{vmatrix}$$

$$= \hat{i}(-6 - 4) - \hat{j}(12 - 16) + \hat{k}(2 + 4)$$

$$\therefore \vec{b}_1 \times \vec{b}_2 = -10\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-10)^2 + 4^2 + 6^2}$$

$$= \sqrt{100 + 16 + 36}$$

$$= \sqrt{152}$$

$$\vec{a}_2 - \vec{a}_1 = (-9 - 6)\hat{i} + (1 - 0)\hat{j} + (6 - 3)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = -15\hat{i} + \hat{j} + 3\hat{k}$$

Now,

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (-10\hat{i} + 4\hat{j} + 6\hat{k}) \cdot (-15\hat{i} + \hat{j} + 3\hat{k})$$

$$= ((-10) \times (-15)) + (4 \times 1) + (6 \times 3)$$

$$= 150 + 4 + 18$$

$$= 172$$

Thus, the shortest distance the given lines is

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\therefore d = \left| \frac{172}{\sqrt{152}} \right|$$

$$\therefore d = \frac{172}{2\sqrt{38}}$$

$$\therefore d = \frac{86}{\sqrt{38}}$$

$$d = \frac{86}{\sqrt{38}} \text{ units}$$

36. i.  $x + 0.21 = 0.44 \Rightarrow x = 0.23$   
 ii.  $0.41 + y + 0.44 + 0.11 = 1 \Rightarrow y = 0.04$   
 iii.  $P\left(\frac{C}{B}\right) = \frac{P(C \cap B)}{P(B)}$

$$P(B) = 0.09 + 0.04 + 0.23 = 0.36$$

$$P\left(\frac{C}{B}\right) = \frac{0.23}{0.36} = \frac{23}{36}$$

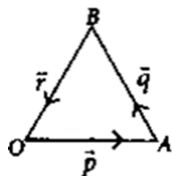
**OR**

$$P(A \text{ or } B \text{ but not } C)$$

$$= 0.32 + 0.09 + 0.04$$

$$= 0.45$$

37. i. Let OAB be a triangle such that



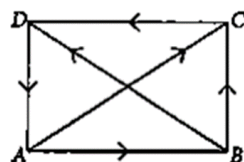
$$\vec{AO} = -\vec{p}, \vec{AB} = \vec{q}, \vec{BO} = \vec{r}$$

$$\text{Now, } \vec{q} + \vec{r} = \vec{AB} + \vec{BO}$$

$$= \vec{AO} = -\vec{p}$$

- ii. From triangle law of vector addition,

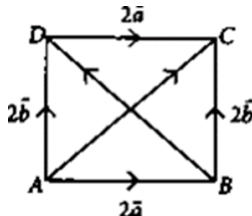
$$\vec{AC} + \vec{BD} = \vec{AB} + \vec{BC} + \vec{BC} + \vec{CD}$$



$$= \vec{AB} + 2\vec{BC} + \vec{CD}$$

$$= \vec{AB} + 2\vec{BC} - \vec{AB} = 2\vec{BC} \quad [\because \vec{AB} = -\vec{CD}]$$

- iii. In  $\triangle ABC$ ,  $\vec{AC} = 2\vec{a} + 2\vec{b} \dots (i)$



$$\text{and in } \triangle ABD, 2\vec{b} = 2\vec{a} + \vec{BD} \dots (ii) \quad [\text{By triangle law of addition}]$$

$$\text{Adding (i) and (ii), we have } \vec{AC} + 2\vec{b} = 4\vec{a} + \vec{BD} + 2\vec{b}$$

$$\Rightarrow \vec{AC} - \vec{BD} = 4\vec{a}$$

**OR**

Since T is the mid point of YZ

$$\text{So, } \vec{YT} = \vec{TZ}$$

$$\text{Now, } \vec{XY} + \vec{XZ} = (\vec{XT} + \vec{TY}) + (\vec{XT} + \vec{TZ}) \quad [\text{By triangle law}]$$

$$= 2\vec{XT} + \vec{TY} + \vec{TZ} = 2\vec{XT} \quad [\because \vec{TY} = -\vec{YT}]$$

38. i. We have,  $I(x) = \frac{1000}{x^2} + \frac{125}{(600-x)^2}$

$$\Rightarrow I'(x) = \frac{-2000}{x^3} + \frac{250}{(600-x)^3} \text{ and}$$

$$\Rightarrow I''(x) = \frac{6000}{x^4} + \frac{750}{(600-x)^4}$$

For maxima/minima,  $I'(x) = 0$

$$\Rightarrow \frac{2000}{x^3} = \frac{250}{(600-x)^3} \Rightarrow 8(600-x)^3 = x^3$$

Taking cube root on both sides, we get

$$2(600 - x) = x \Rightarrow 1200 = 3x \Rightarrow x = 400$$

Thus,  $I(x)$  is minimum when you are at 400 feet from the strong intensity lamp post.

ii. At a distance of 200 feet from the weaker lamp post.

Since  $I(x)$  is minimum when  $x = 400$  feet, therefore the darkest spot between the two light is at a distance of 400 feet from a stronger lamp post, i.e., at a distance of  $600 - 400 = 200$  feet from the weaker lamp post.

iii.  $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$

Since, the distance is  $x$  feet from the stronger light, therefore the distance from the weaker light will be  $600 - x$ .

So, the combined light intensity from both lamp posts is given by  $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$ .

**OR**

We know that  $I(x) = \frac{1000}{x^2} + \frac{125}{(600-x)^2}$

When  $x = 400$

$$I(x) = \frac{1000}{160000} + \frac{125}{(600-400)^2}$$

$$= \frac{1}{160} + \frac{125}{40000} = \frac{1}{160} + \frac{1}{320} = \frac{3}{320} \text{ units}$$