

Class XII Session 2025-26

Subject - Mathematics

Sample Question Paper - 4

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. Let $A = \{1, 2, 3\}$ and let $R = \{(1,1), (2, 2), (3, 3), (1, 2), (2,1), (2, 3), (3, 2)\}$. Then, R is [1]
a) reflexive and symmetric but not transitive b) an equivalence relation
c) symmetric and transitive but not reflexive d) reflexive and transitive but not symmetric
2. The value of $\sin^{-1}(\cos \frac{\pi}{9})$ is [1]
a) $\frac{5\pi}{9}$ b) $\frac{7\pi}{18}$
c) $\frac{-5\pi}{9}$ d) $\frac{\pi}{9}$
3. For which value of x , are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal? [1]
a) ± 2 b) 2
c) ± 3 d) -3
4. The function $f(x) = |\cos x|$ is [1]
a) everywhere continuous but not differentiable at $(2n + 1) \frac{\pi}{2}, n \in \mathbb{Z}$ b) either continuous or differentiable at $(2n + 1) \frac{\pi}{2}, n \in \mathbb{Z}$
c) neither continuous nor differentiable at $(2n + 1) \frac{\pi}{2}, n \in \mathbb{Z}$ d) everywhere continuous and differentiable
5. If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x -axis and y -axis, then the angle which it [1]

makes with positive z-axis is:

- a) 0
b) $\frac{3\pi}{4}$
c) $\frac{\pi}{2}$
d) $\frac{\pi}{4}$

6. Consider the following statements in respect of the differential equation $\frac{d^2y}{dx^2} + \cos\left(\frac{dy}{dx}\right) = 0$ [1]

- i. The degree of the differential equation is not defined.
ii. The order of the differential equation is 2.

Which of the above statement(s) is/are correct?

- a) Neither (i) nor (ii)
b) Both (i) and (ii)
c) Only (ii)
d) Only (i)

7. The graph of the inequality $2x + 3y > 6$ is [1]

- a) whole XOY – plane excluding the points on the line $2x + 3y = 6$.
b) half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.
c) entire XOY plane.
d) half plane that contains the origin.

8. Magnitude of the vector $\vec{a} = 2\hat{i} - 7\hat{j} - 3\hat{k}$ is [1]

- a) $\sqrt{61}$
b) $\sqrt{65}$
c) $\sqrt{62}$
d) $\sqrt{63}$

9. $\int \frac{dx}{(2-3x)} = ?$ [1]

- a) $-3 \log |2 - 3x| + C$
b) $\log |2 + 3x| + C$
c) $-\log |2 - 3x| + C$
d) $-\frac{1}{3} \log |2 - 3x| + C$

10. The order of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & -1 \end{bmatrix}$ is _____. [1]

- a) 2×3
b) 3×3
c) 2×2
d) 3×2

11. The point at which the maximum value of $x + y$, subject to the constraints $x + 2y \leq 70$, $2x + y \leq 95$, $x, y \geq 0$ is obtained, is [1]

- a) (30, 25)
b) (20, 35)
c) (35, 20)
d) (40, 15)

12. For what value of λ , the projection of vector $\hat{i} + \lambda\hat{j}$ on vector $\hat{i} - \hat{j}$ is $\sqrt{2}$? [1]

- a) 0
b) -1
c) 1
d) 3

13. $\begin{vmatrix} \cos 70^\circ & \sin 20^\circ \\ \sin 70^\circ & \cos 20^\circ \end{vmatrix} = ?$ [1]

- a) $\sin 50^\circ$
b) 0
c) $\cos 50^\circ$
d) 1

14. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the the probability that exactly two of the three balls were red, the first ball being red, is [1]

a) $\frac{4}{7}$

b) $\frac{5}{28}$

c) $\frac{15}{28}$

d) $\frac{1}{3}$

15. General solution of $y \log y \, dx - x \, dy = 0$

[1]

a) $y = e^{cx}$

b) $y^2 = e^{cx}$

c) $y = e^{cx} + e^{-cx}$

d) $y = e^{-cx}$

16. Let L denote the set of all straight lines in a plane. Let a relation R be defined by lRm if and only if l is perpendicular to $m \, \forall l, m \in L$. Then R is

[1]

a) reflexive

b) symmetric

c) Asymmetric

d) transitive

17. For what value of k may the function $f(x) = \begin{cases} k(3x^2 - 5x), & x \leq 0 \\ \cos x, & x > 0 \end{cases}$ become continuous?

[1]

a) 1

b) $-\frac{1}{2}$

c) 0

d) No value

18. The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is

[1]

a) perpendicular to z -axisb) parallel to x -axisc) parallel to y -axisd) parallel to z -axis19. **Assertion (A):** The rate of change of area of a circle with respect to its radius r when $r = 6$ cm is 12π cm²/cm.

[1]

Reason (R): Rate of change of area of a circle with respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle.a) Both A and R are true and R is the correct explanation of A .b) Both A and R are true but R is not the correct explanation of A .c) A is true but R is false.d) A is false but R is true.20. The value of λ for which the two vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} + \lambda\hat{j} + \hat{k}$ are perpendicular is

[1]

a) 8

b) 4

c) 6

d) 2

Section B21. Evaluate: $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

[2]

OR

Write the interval for the principal value of function and draw its graph: $\tan^{-1} x$.22. Find all the points of discontinuity of the greatest integer function defined by $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x .

[2]

23. Find the intervals in which $f(x) = (x+2)e^{-x}$ is increasing or decreasing.

[2]

OR

If $x^{30}y^{20} = (x+y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.24. Evaluate: $\int e^x (\cot x - \operatorname{cosec}^2 x) \, dx$

[2]

25. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, find AA^T

[2]

Section C26. Using integration, find the area of the region bounded by the lines $x - y = 0$, $3x - y = 0$ and $x + y = 12$.

[3]

27. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure. [3]

28. Evaluate the definite integral $\int_0^\pi \frac{1}{1+\sin x} dx$ [3]

OR

Evaluate: $\int \frac{1}{5-4\cos x} dx$

29. Find the general solution of the differential equation: $(x^2 + 1) \frac{dy}{dx} - 2xy = (x^2 + 1)(x^2 + 2)$ [3]

OR

Solve the differential equation: $[x\sqrt{x^2 + y^2} - y^2] dx + xy dy = 0$

30. Solve the Linear Programming Problem graphically: [3]

Maximize $Z = x + y$ Subject to

$$-2x + y \leq 1$$

$$x \leq 2$$

$$x + y \leq 3$$

$$x, y \geq 0$$

OR

Determine graphically the minimum value of the objective function $Z = -50x + 20y$ subject to the constraints:

$$2x - y \geq -5$$

$$3x + y \geq 3$$

$$2x - 3y \leq 12$$

$$x \geq 0, y \geq 0$$

31. Show that the function $f(x)$ defined by $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x < 0 \end{cases}$ is continuous at $x = 0$. [3]

Section D

32. Evaluate: $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$. [5]

33. Let $A = \mathbb{R} - \{3\}$, $B = \mathbb{R} - \{1\}$. If $f: A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3} \forall x \in A$. Then, show that f is bijective. [5]

OR

Let $A = \{1, 2, 3, \dots, 9\}$ and R be the relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalence class $[(2, 5)]$.

34. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find the value of A^{-1} . [5]

Using A^{-1} , solve the system of linear equations:

$$x - 2y = 10,$$

$$2x - y - z = 8,$$

$$-2y + z = 7$$

35. Show that the lines $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$ intersect and find their point of intersection. [5]

OR

Find the length shortest distance between the lines: $\frac{x-3}{3} = \frac{y-8}{-1} = z - 3$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

Section E

36. Read the following text carefully and answer the questions that follow: [4]

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's

selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.



- Find the probability that both of them are selected. (1)
- The probability that none of them is selected. (1)
- Find the probability that only one of them is selected. (2)

OR

Find the probability that atleast one of them is selected. (2)

37. **Read the following text carefully and answer the questions that follow:**

[4]

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association).

Let A be a 1×2 matrix and B be a 2×1 matrix, representing the investment and interest rate on each bond respectively.



Based on the above information, answer the following questions.

- If ₹ 15000 is invested in bond X, then what is the matrix representation of A and B? (1)
- If ₹ 15,000 is invested in bond X, how can we determine the total amount of interest received on both bonds? (1)
- How much is the investment in two bonds if the trust fund obtains an annual total interest of ₹3200? (2)

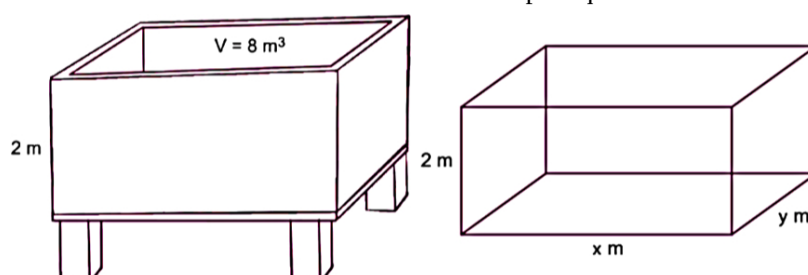
OR

What is the amount of investment in bond Y if the interest given to the old age home is ₹500? (2)

38. **Read the following text carefully and answer the questions that follow:**

[4]

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m^3 as shown below. The construction of the tank costs ₹70 per sq. metre for the base and ₹45 per square metre for sides.



- i. Express making cost C in terms of length of rectangle base. (1)
- ii. If x and y represent the length and breadth of its rectangular base, then find the relation between the variables. (1)
- iii. Find the value of x so that the cost of construction is minimum. (2)

OR

Verify by second derivative test that cost is minimum at a critical point. (2)

Solution

Section A

1. (a) reflexive and symmetric but not transitive

Explanation:

reflexive and symmetric but not transitive .

Reflexivity and transitivity follows from definition.

Here, (3,2) ,(2,1) are in R but (3,1) is not in R, so R is not transitive.

- 2.

(b) $\frac{7\pi}{18}$

Explanation:

$$\sin^{-1}(\cos \frac{\pi}{9}) = \sin^{-1}(\sin(\frac{\pi}{2} - \frac{\pi}{9})) = \sin^{-1}(\sin \frac{7\pi}{18}) = \frac{7\pi}{18}$$

3. (a) ± 2

Explanation:

$$\pm 2$$

$$\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$$

$$2x^2 + 15 = 20 + 3$$

$$2x^2 = 23 - 15$$

$$2x^2 = 8$$

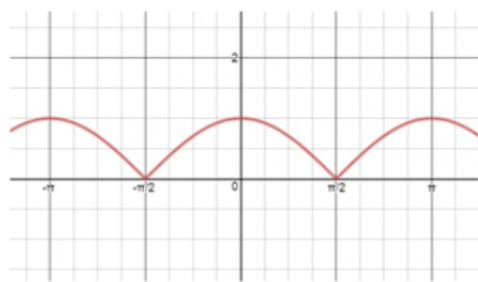
$$x^2 = 4$$

$$x = \pm 2$$

4. (a) everywhere continuous but not differentiable at $(2n + 1) \frac{\pi}{2}$, $n \in \mathbb{Z}$

Explanation:

Given that $f(x) = |\cos x|$



From the graph it is evident that it is everywhere continuous but not differentiable at $(2n + 1) \frac{\pi}{2}$, $n \in \mathbb{Z}$

- 5.

(c) $\frac{\pi}{2}$

Explanation:

$$\frac{\pi}{2}$$

- 6.

(b) Both (i) and (ii)

Explanation:

Both (i) and (ii)

- 7.

(b) half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.

Explanation:

The inequality $2x + 3y > 6$ represents a half-plane where points satisfy the condition of being above the line $2x + 3y = 6$. The line $2x + 3y = 6$ is not included in the solution, since the inequality is strict (" $>$ " and not " $\geq$ ").

To determine which half-plane, we can test a point not on the line. Testing the origin $(0, 0)$: Substitute $x = 0$ and $y = 0$ into the inequality: $2(0) + 3(0) = 0$ Since $0 \not> 6$, the origin is not in the solution region.

Thus, the solution is a half-plane that does not contain the origin nor the points on the line.

The correct option is: half-plane that neither contains the origin nor the points of the line $2x + 3y = 6$.

8.

(c) $\sqrt{62}$

Explanation:

We have:

$$\vec{a} = 2\hat{i} - 7\hat{j} - 3\hat{k},$$

then,

$$|\vec{a}| = \sqrt{2^2 + (-7)^2 + (-3)^2} = \sqrt{4 + 49 + 9} = \sqrt{62}.$$

9.

(d) $-\frac{1}{3}\log|2 - 3x| + C$

Explanation:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x^2} dx = \log x + c$

Therefore ,

Put $2 - 3x = t$

$$-3 dx = dt$$

$$= \frac{1}{3} \int \frac{1}{t} dt$$

$$= -\frac{1}{3} \log t + c$$

$$= -\frac{1}{3} \log |2 - 3x| + c$$

10. (a) 2×3

Explanation:

Order of a matrix is given by

(number of rows) $\times$ (number of columns)

$\therefore$ Order of matrix A = 2×3

11.

(d) (40,15)

Explanation:

We need to maximize the function $z = x + y$ Converting the given inequations into equations, we obtain

$$x + 2y = 70, 2x + y = 95, x \geq 0 \text{ and } y \geq 0$$

Region represented by $x + 2y \leq 70$:

The line $x + 2y = 70$ meets the coordinate axes at A(70, 0) and B(0, 35) respectively. By joining these points we obtain the line $x + 2y = 70$. Clearly (0, 0) satisfies the inequation $x + 2y \leq 70$. So, the region containing the origin represents the solution set of the inequation $x + 2y \leq 70$.

Region represented by $2x + y \leq 95$:

The line $2x + y = 95$ meets the coordinate axes at $C\left(\frac{95}{2}, 0\right)$ respectively. By joining these points we

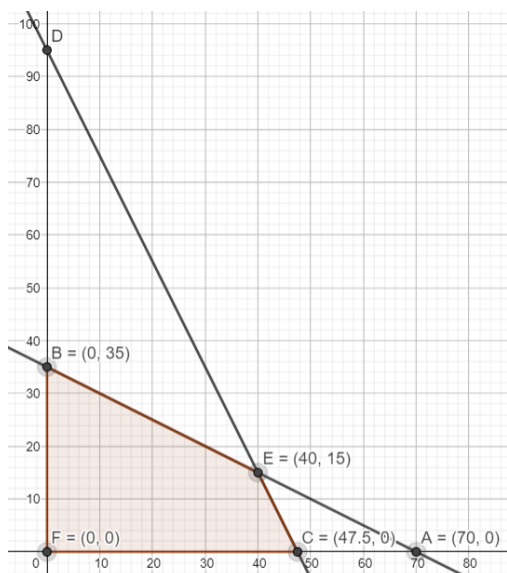
obtain the line $2x + y = 95$

Clearly (0, 0) satisfies the inequation $2x + y \leq 95$. So, the region containing the origin represents the solution set of the inequation $2x + y \leq 95$

Region represented by $x \geq 0$ and $y \geq 0$:

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$, and $y \geq 0$

The feasible region determined by the system of constraints $x + 2y \leq 70$, $2x + y \leq 95$, $x \geq 0$, and $y \geq 0$ are as follows.



The corner points of the feasible region are $O(0, 0)$, $C(\frac{95}{2}, 0)$, $E(40, 15)$ and $B(0, 35)$.

The value of Z at these corner points are as follows.

Corner point : $z = x + y$

$O(0, 0) : 0 + 0 = 0$

$C(\frac{95}{2}, 0) : \frac{95}{2} + 0 = \frac{95}{2}$

$E(40, 15) : 40 + 15 = 55$

$B(0, 35) : 0 + 35 = 35$

We see that maximum value of the objective function Z is 55 which is at $(40, 15)$.

12.

(b) -1

Explanation:

-1

13.

(b) 0

Explanation:

By evaluating given determinant and using $\sin(90^\circ - A) = \cos A$, we get value of $\det. = 0$

14.

(a) $\frac{4}{7}$

Explanation:

Let E_1 = Event that first ball is red = (RRR, RRB, RBR, RBB)

And E_2 = Event that exactly two of three balls being red = (RRR, RRB)

$$\begin{aligned} \therefore P(E_1) &= P_R \cdot P_R \cdot P_R + P_R \cdot P_R \cdot P_B + P_R \cdot P_B \cdot P_R + P_R \cdot P_B \cdot P_B \\ &= \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} + \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} + \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{4}{6} + \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} \\ &= \frac{60+60+60+30}{336} = \frac{210}{336} \end{aligned}$$

$$\begin{aligned} P(E_1 \cap E_2) &= P_B \cdot P_R \cdot P_R + P_R \cdot P_R \cdot P_R \\ &= \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{4}{6} + \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{120}{336} \end{aligned}$$

$$\therefore P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{120/336}{210/336} = \frac{4}{7}$$

15.

(a) $y = e^{cx}$

Explanation:

$$y \log y \, dx = x \, dy$$

$$\int \frac{1}{x} dx = \int \frac{1}{y \log y} dy$$

$$\log|x| = \log|\log y| + \log C \quad \text{Since } \int \frac{f'(x)dx}{f(x)} = \log|f(x)| + c \text{ and } \frac{1}{C} = c \text{ a new constant}$$

$$\log x = \log(C \log y)$$

$$x = C \log y$$

$$\log y = \frac{1}{C} x$$

$$\log y = cx$$

$$y = e^{cx}$$

16.

(b) symmetric

Explanation:

Let $(x, y) \in R$, such that $x \perp y$.

We can also write from above that, $y \perp x$.

Hence, $(y, x) \in R$

So, They are symmetric.

17.

(d) No value

Explanation:

No value

18. **(a)** perpendicular to z-axis

Explanation:

We have,

$$\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$$

Also, the given line is parallel to the vector $\vec{b} = 3\hat{i} + \hat{j} + 0\hat{k}$

Let $x\hat{i} + y\hat{j} + z\hat{k}$ be perpendicular to the given line.

Now,

$$3x + 4y + 0z = 0$$

It is satisfied by the coordinates of z-axis, i.e. $(0, 0, 1)$

Hence, the given line is perpendicular to z-axis.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20. **(a)** 8

Explanation:

We know that, dot product of two orthogonal vectors is always 0.

Hence,

$$(2\hat{i} - \hat{j} + 2\hat{k}) \cdot (3\hat{i} + \lambda\hat{j} + \hat{k}) = 0$$

$$\Rightarrow 6 - \lambda + 2 = 0$$

$$\Rightarrow \lambda = 8$$

Section B

$$21. \tan^{-1} \sqrt{3} - \sec^{-1}(-2) = \tan^{-1} \sqrt{3} - [\pi - \sec^{-1} 2]$$

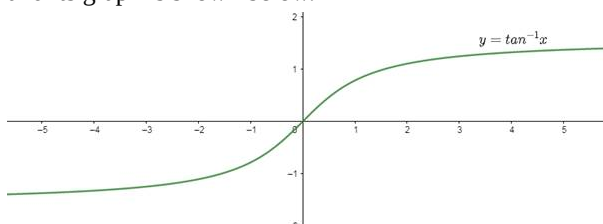
$$= \frac{\pi}{3} - \pi + \cos^{-1} \left(\frac{1}{2} \right)$$

$$= -\frac{2\pi}{3} + \frac{\pi}{3} = -\frac{\pi}{3}$$

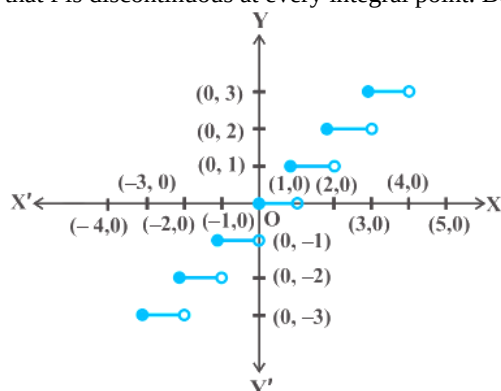
OR

Principal value branch of $\tan^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$

and its graph is shown below.



22. First, observe that f is defined for all real numbers. The graph of the function is given in the figure. From the graph, it looks like that f is discontinuous at every integral point. Below we explore if this is true.



Case 1: Let c be a real number which is not equal to any integer. It is evident from the graph that for all real numbers close to c the value of the function is equal to $[c]$; i.e., $\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^-} [x] = [c]$. Also $f(c) = [c]$ and hence the function is continuous at all real numbers not equal to integers.

Case 2: Let c be an integer. Then we can find a sufficiently small real number $r > 0$ such that $[c - r] = c - 1$ whereas $[c + r] = c$. This, in terms of limits mean that

$$\lim_{x \rightarrow c^-} f(x) = c - 1, \quad \lim_{x \rightarrow c^+} f(x) = c$$

Since these limits cannot be equal to each other for any c , the function is discontinuous at every integral point.

23. Given: $f(x) = (x + 2)e^{-x}$

$$f'(x) = e^{-x} - e^{-x}(x+2)$$

$$= e^{-x}(1 - x - 2)$$

$$= -e^{-x}(x+1)$$

For Critical points

$$f'(x) = 0$$

$$\Rightarrow -e^{-x}(x+1) = 0$$

$$\Rightarrow x = -1$$

$$\text{Clearly } f'(x) > 0 \text{ if } x < -1$$

$$f'(x) < 0 \text{ if } x > -1$$

Hence $f(x)$ increases in $(-\infty, -1)$, decreases in $(-1, \infty)$

OR

Taking log of both sides, we get

$$30 \log x + 20 \log y = 50 \log(x + y)$$

Differentiating both sides w.r.t. x , we get

$$\frac{30}{x} + \frac{20}{y} \frac{dy}{dx} = \frac{50}{x+y} \left(1 + \frac{dy}{dx} \right)$$

$$\Rightarrow \frac{dy}{dx} \left(\frac{20x - 30y}{y(x+y)} \right) = \frac{20x - 30y}{x(x+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

24. Let $I = \int e^x (\cot x - \operatorname{cosec}^2 x) dx$

Here, $f(x) = \cot x$ put $e^x f(x) = t$

$$f'(x) = -\operatorname{cosec}^2 x$$

$$\text{let } e^x \cot x = t$$

Diff. both sides w.r.t x

$$e^x \cot x + e^x (-\operatorname{cosec}^2 x) = \frac{dt}{dx}$$

$$\Rightarrow e^x (\cot x - \operatorname{cosec}^2 x) = \frac{dt}{dx}$$

$$\therefore \int e^x (\cot x - \operatorname{cosec}^2 x) dx = \int dt$$

$$= t + C [e^x \cot x = t]$$

$$= e^x \cot x + C$$

25. Given:

$$A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$

To find the matrix as a result of the product AA^T

Firstly, we find the A^T (which is the transpose of the matrix A)

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2×2 matrix, then the transpose of a matrix is $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

So,

$$A^T = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\therefore AA^T = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \quad (\text{Using the rule of matrix multiplication we get})$$

$$= \begin{bmatrix} \cos x \times \cos x + (-\sin x) \times (-\sin x) & \sin x \times \cos x + (-\sin x) \times \cos x \\ \sin x \times \cos x + \cos x \times (-\sin x) & \sin x \times \sin x + \cos x \times \cos x \end{bmatrix}$$

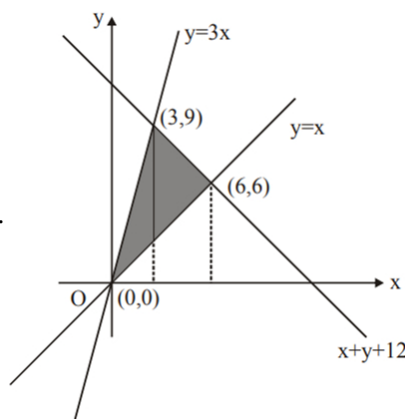
$$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin x \cos x - \sin x \cos x \\ \sin x \cos x - \sin x \cos x & \sin^2 x + \cos^2 x \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad [\because \cos^2 x + \sin^2 x = 1 \text{ using the property of the trigonometry identity.}]$$

$$\Rightarrow AA^T = I_2$$

Section C

26.



$$\text{Required area} = \int_0^3 3x dx + \int_3^6 (12 - x) dx - \int_0^6 x dx$$

$$= 3 \left[\frac{x^2}{2} \right]_0^3 + \left[12x - \frac{x^2}{2} \right]_3^6 - \left[\frac{x^2}{2} \right]_0^6$$

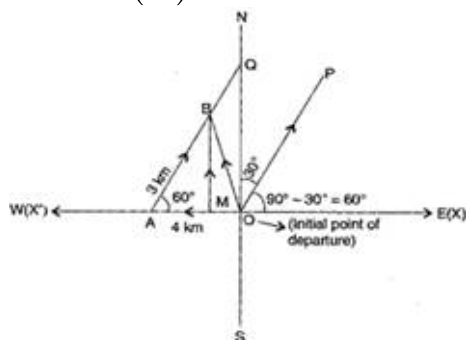
$$= \frac{27}{2} + \frac{45}{2} - 18 = 18 \text{ sq units}$$

27. Let the initial point of departure is origin (0, 0) and the girl walks a distance OA = 4 km towards west.

Through the point A, draw a line AQ parallel to a line OP, which is 30° East of North, i.e., in East-North quadrant making an angle of 30° with North.

Again, let the girl walks a distance AB = 3 km along this direction $\overrightarrow{OQ}$

$$\therefore \overrightarrow{OA} = 4(-\hat{i}) = -4\hat{i} \quad \dots (i) \quad [\because \text{Vector } \overrightarrow{OA} \text{ is along OX}']$$



Now, draw BM perpendicular to x - axis.

In $\triangle AMB$ by Triangle Law of Addition of vectors,

$$\overrightarrow{AB} = \overrightarrow{AM} + \overrightarrow{MB} = (AM)\hat{i} + (MB)\hat{j}$$

Dividing and multiplying by AB in R.H.S.,

$$\vec{AB} = AB \frac{AM}{AB} \hat{i} + AB \frac{MB}{AB} \hat{j} = 3 \cos 60^\circ \hat{i} + 3 \sin 60^\circ \hat{j}$$

$$\Rightarrow AB = 3 \frac{1}{2} \hat{i} + 3 \frac{\sqrt{3}}{2} \hat{j} = \frac{3}{2} \hat{i} + \frac{3\sqrt{3}}{2} \hat{j} \dots (ii)$$

∴ Girl's displacement from her initial point O of departure to final point B,

$$\vec{OB} = \vec{OA} + \vec{AB} = -4\hat{i} + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j} \right) = \left(-4 + \frac{3}{2} \right) \hat{i} + \frac{3\sqrt{3}}{2} \hat{j}$$

$$\Rightarrow \vec{OB} = \frac{-5}{2} \hat{i} + \frac{3\sqrt{3}}{2} \hat{j}$$

28. Let

$$I = \int_0^x \frac{1}{1+\sin x} dx$$

Multiplying Numerator and Denominator of the integrand by (1-sin x), gives

$$I = \int_0^x \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx$$

$$= \int_0^x \frac{(1-\sin x)}{(1^2-\sin^2 x)} dx$$

$$= \int_0^x \frac{1-\sin x}{(\cos^2 x)} dx$$

$$= \int_0^x \frac{1}{\cos^2 x} dx - \int_0^x \frac{\sin x}{\cos^2 x} dx$$

$$= \int_0^x \sec^2 x dx - \int_0^x \tan x \cdot \sec x dx$$

$$I = [\tan x]_0^x - [\sec x]_0^x$$

$$= [\tan \pi - \tan 0] - [\sec \pi - \sec 0]$$

$$= [0 - 0] - [-1 - 1]$$

$$= 2$$

$$\therefore \int_0^x \frac{1}{1+\sin x} dx = 2$$

OR

Let the given integral be,

$$I = \int \frac{1}{5-4 \cos x} dx$$

$$\text{Putting } \cos x = \frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}}$$

$$\Rightarrow I = \int \frac{1}{5-4 \left(\frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}} \right)} dx$$

$$= \int \frac{(1+\tan^2 \frac{x}{2})}{5(1+\tan^2 \frac{x}{2})-4+4 \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \left(\frac{x}{2} \right)}{9 \tan^2 \frac{x}{2} + 1} dx$$

$$\text{Let } \tan \left(\frac{x}{2} \right) = t$$

$$\Rightarrow \frac{1}{2} \sec^2 \left(\frac{x}{2} \right) dx = dt$$

$$\sec^2 \left(\frac{x}{2} \right) dx = 2dt$$

$$\therefore I = \int \frac{2dt}{1-t^2-2t}$$

$$= 2 \int \frac{-2dt}{t^2+2t-1}$$

$$= 2 \int \frac{-2dt}{t^2+2t+1-2}$$

$$= \frac{2}{9} \times 3 \tan^{-1} \left(\frac{t}{\frac{1}{3}} \right) + C$$

$$= \frac{2}{3} \tan^{-1}(3t) + C$$

$$= \frac{2}{3} \tan^{-1} \left(3 \tan \frac{x}{2} \right) + C$$

29. The given differential equation is,

$$(x^2 + 1) \frac{dy}{dx} - 2xy = (x^2 + 1)(x^2 + 2)$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{-2x}{x^2+1} \right) y = x^2 + 2$$

This is of the form $\frac{dy}{dx} + Py = Q$, where

$$P = \frac{-2x}{x^2+1} \text{ and } Q = x^2 + 2$$

Thus the given differential equation is linear differential equation

$$\begin{aligned}
\text{Now, } IF &= e^{\int P dx} \\
&= e^{\int \frac{-2x}{x^2+1} dx} \\
&= e^{-\log(x^2+1)} = (x^2 + 1)^{-1} = \frac{1}{x^2+1}
\end{aligned}$$

Therefore the solution is given by

$$\begin{aligned}
(IF) \cdot y &= \int (IF)Q + C \\
\Rightarrow \frac{1}{x^2+1} \cdot y &= \int \frac{1}{x^2+1} (x^2 + 2) dx + C \\
\Rightarrow \frac{y}{x^2+1} &= \int \frac{x^2+2}{x^2+1} dx + C \\
\Rightarrow \frac{y}{x^2+1} &= \int \frac{x^2+1+1}{x^2+1} dx + C \\
\Rightarrow \frac{y}{x^2+1} &= \int \left\{ 1 + \frac{1}{x^2+1} \right\} dx + C \\
\Rightarrow \frac{y}{x^2+1} &= x + \tan^{-1} x + C \\
\Rightarrow y &= (x^2 + 1)(x + \tan^{-1} x + C)
\end{aligned}$$

OR

The given differential equation is,

$$\begin{aligned}
&[x\sqrt{x^2 + y^2} - y^2] dx + xy dy = 0 \\
\frac{dy}{dx} &= \frac{y^2 - x\sqrt{x^2 + y^2}}{xy}
\end{aligned}$$

This is a homogeneous differential equation

Putting $y = vx$ and $\frac{dy}{dx} = v + x \frac{dv}{dx}$, we get

$$\begin{aligned}
v + x \frac{dv}{dx} &= \frac{v^2 x^2 - x\sqrt{x^2 + v^2 x^2}}{vx^2} \\
\Rightarrow v + x \frac{dv}{dx} &= \frac{v^2 - \sqrt{1+v^2}}{v} \\
\Rightarrow v + x \frac{dv}{dx} &= v - \frac{\sqrt{1+v^2}}{v} \\
\Rightarrow x \frac{dv}{dx} &= \frac{-\sqrt{1+v^2}}{v} \\
\Rightarrow \frac{v}{\sqrt{1+v^2}} dv &= -\frac{1}{x} dx
\end{aligned}$$

Putting $1 + v^2 = t$, we get

$$\begin{aligned}
v dv &= \frac{dt}{2} \\
\therefore \frac{1}{2\sqrt{t}} dt &= -\frac{1}{x} dx
\end{aligned}$$

Integrating both sides, we get

$$\begin{aligned}
\int \frac{1}{2\sqrt{t}} dt &= -\int \frac{1}{x} dx \\
\Rightarrow \sqrt{t} &= -\log|x| + \log C \dots (i)
\end{aligned}$$

Substituting the value of t in (i), we get

$$\sqrt{1 + v^2} = \log \left| \frac{C}{x} \right|$$

Hence, $\sqrt{y^2 + x^2} = x \log \left| \frac{C}{x} \right|$ is the required solution.

30. We need to maximize $z = x + y$

First, we will convert the given inequations into equations, we obtain the following equations:

$$-2x + y = 1, x = 2, x + y = 3, x = 0 \text{ and } y = 0$$

The line $-2x + y = 1$ meets the coordinate axis at $A\left(-\frac{1}{2}, 0\right)$ and $B(0, 1)$. Join these points to obtain the line $-2x + y = 1$.

Clearly, $(0, 0)$ satisfies the inequation $-2x + y \leq 1$. So, the region in xy -plane that contains the origin represents the solution set of the given equation.

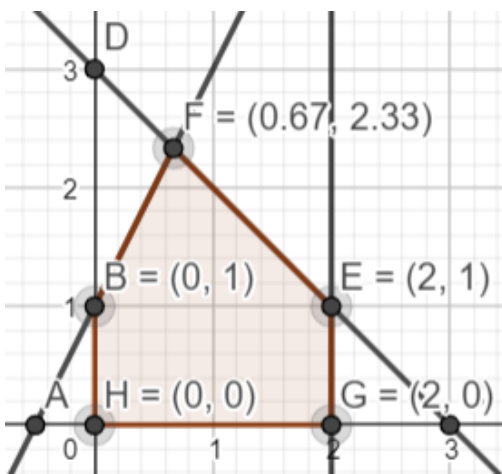
$x = 2$ is the line passing through $(2, 0)$ and parallel to the Y axis.

The region below the line $x = 2$ will satisfy the given inequation. The line $x + y = 3$ meets the coordinate axis at $C(3, 0)$ and $D(0, 3)$. Join these points to obtain the line $x + y = 3$.

Clearly, $(0, 0)$ satisfies the inequation $x + y \leq 3$. So, the region in xy -plane that contains the origin represents the solution set of the given equation.

Region represented by $x \geq 0$ and $y \geq 0$ (non-negative restrictions)

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations. These lines are drawn using a suitable scale.



The corner points of the feasible region are $O(0,0)$, $G(2,0)$, $E(2,1)$ and $F\left(\frac{2}{3}, \frac{7}{3}\right)$

The values of objective function at the corner points are as follows:

Corner point : $Z = x + y$

$O(0, 0) : 0 + 0 = 0$

$C(2, 0) : 2 + 0 = 2$

$E(2, 1) : 2 + 1 = 3$

$F\left(\frac{2}{3}, \frac{7}{3}\right) : \frac{2}{3} + \frac{7}{3} = \frac{9}{3} = 3$

We see that the maximum value of the objective function z is 3 which is at $E(2,1)$ and $F\left(\frac{2}{3}, \frac{7}{3}\right)$

Thus, the optimal value of objective function z is 3.

OR

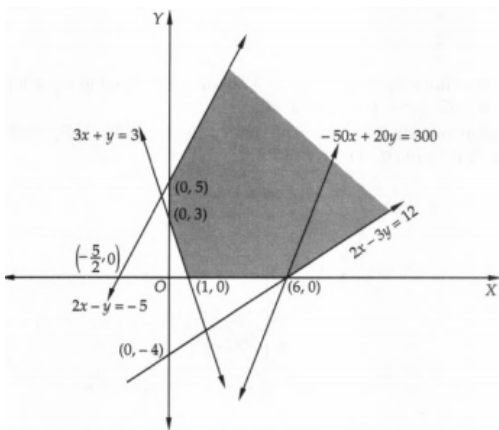
$$2x - y \geq -5$$

$$3x + y \geq 3$$

$$2x - 3y \leq 12$$

$$x \geq 0, y \geq 0$$

The feasible region of the system of inequations given in constraints is shown in a figure. We observe that the feasible region is unbounded.



The values of the objective function Z at the corner points are given in the following table:

Corner point (x, y)	Value of the objective function $Z = -50x + 20y$
(0,5)	$Z = -50 \times 0 + 20 \times 5 = 100$
(0,3)	$Z = -50 \times 0 + 20 \times 3 = 60$
(1,0)	$Z = -50 \times 1 + 20 \times 0 = -50$
(6,0)	$Z = -50 \times 6 + 20 \times 0 = -300$

Clearly, -300 is the smallest value of Z at the corner point (6, 0). Since the feasible region is unbounded, therefore, to check whether -300 is the minimum value of Z , we draw the line $-300 = -50x + 20y$ and check whether the open half plane $-50x + 20y < -300$ has points in common with the feasible region or not. From Fig., we find that the open half plane represented by $-50x + 20y < -300$ has points in common with the feasible region. Therefore, $Z = -50x + 20y$ has no minimum value subject to the given constraints.

31. To show that the given function is continuous at $x = 0$, we show that

$$(\text{LHL})_{x=0} = (\text{RHL})_{x=0} = f(0) \dots (i)$$

$$\text{Here, we have } f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x < 0 \end{cases}$$

$$\text{Now, LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{4(1-\sqrt{1-x})}{x}$$

$$= \lim_{h \rightarrow 0} \frac{4[1-\sqrt{1-(0-h)}]}{0-h}$$

$$= \lim_{h \rightarrow 0} \frac{4[1-\sqrt{1+h}]}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{4[1-\sqrt{1+h}]}{-h} \times \frac{1+\sqrt{1+h}}{1+\sqrt{1+h}}$$

$$= \lim_{h \rightarrow 0} \frac{4[(1)^2 - (\sqrt{1+h})^2]}{-h[1+\sqrt{1+h}]}$$

$$= \lim_{h \rightarrow 0} \frac{4[1-(1+h)]}{-h[1+\sqrt{1+h}]}$$

$$= \lim_{h \rightarrow 0} \frac{-h \times 4}{-h[1+\sqrt{1+h}]}$$

$$= \lim_{h \rightarrow 0} \frac{4}{1+\sqrt{1+h}}$$

$$= \frac{4}{1+\sqrt{1}} = \frac{4}{2} = 2$$

$$\text{and RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \left(\frac{\sin x}{x} + \cos x \right)$$

$$\Rightarrow \text{RHL} = \lim_{h \rightarrow 0} \left(\frac{\sin h}{h} + \cos h \right)$$

$$= \lim_{h \rightarrow 0} \frac{\sin h}{h} + \lim_{h \rightarrow 0} \cos h$$

$$= 1 + \cos 0$$

$$= 1 + 1$$

$$= 2$$

Also, given that $x = 0$, $f(x) = 2 \Rightarrow f(0) = 2$

Since, $(\text{LHL})_{x=0} = (\text{RHL})_{x=0} = f(0) = 2$

Therefore, $f(x)$ is continuous at $x = 0$.

Section D

32. We have

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = \frac{(t+1)(t+2)}{(t+3)(t+4)}, \text{ where } x^2 = t$$

$$= \frac{(t^2+3t+2)}{(t^2+7t+12)} = 1 - \frac{(4t+10)}{(t+3)(t+4)}$$

Use partial fractions for 2nd part

$$\text{Let } \frac{(4t+10)}{(t+3)(t+4)} = \frac{A}{(t+3)} + \frac{B}{(t+4)}$$

$$\Rightarrow (4t+10) = A(t+4) + B(t+3) \dots (i)$$

Putting $t = -3$ in (i), we get $A = -2$

Putting $t = -4$ in (i), we get $B = 6$

$$\therefore \frac{(4t+10)}{(t+3)(t+4)} = \frac{-2}{(t+3)} + \frac{6}{(t+4)} \dots (ii)$$

$$\text{Thus, } \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = \frac{(t+1)(t+2)}{(t+3)(t+4)}, \text{ where } x^2 = t$$

$$= \frac{(t^2+3t+2)}{(t^2+7t+12)} = 1 - \frac{(4t+10)}{(t+3)(t+4)}$$

$$= 1 - \left\{ \frac{-2}{(t+3)} + \frac{6}{(t+4)} \right\} \text{ [from (ii)]}$$

$$= \left\{ 1 + \frac{2}{(t+3)} - \frac{6}{(t+4)} \right\}$$

$$= \left\{ 1 + \frac{2}{(x^2+3)} - \frac{6}{(x^2+4)} \right\}$$

$$\therefore \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx = \int \left\{ 1 + \frac{2}{(x^2+3)} - \frac{6}{(x^2+4)} \right\} dx$$

$$= \int dx + 2 \int \frac{dx}{(x^2+3)} - 6 \int \frac{dx}{(x^2+4)}$$

$$= x + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \frac{6}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= x + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - 3 \tan^{-1}\left(\frac{x}{2}\right) + C$$

33. Given that, $A = \mathbb{R} - \{3\}$, $B = \mathbb{R} - \{1\}$.

$f: A \rightarrow B$ is defined by $f(x) = \frac{x-2}{x-3} \forall x \in A$

For injectivity

$$\text{Let } f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$$

$$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1$$

$$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$$

So, $f(x)$ is an injective function

For surjectivity

$$\text{Let } y = \frac{x-2}{x-3} \Rightarrow x - 2 = xy - 3y$$

$$\Rightarrow x(1 - y) = 2 - 3y \Rightarrow x = \frac{2-3y}{1-y}$$

$$\Rightarrow x = \frac{3y-2}{y-1} \in A, \forall y \in B \text{ [codomain]}$$

So, $f(x)$ is surjective function.

Hence, $f(x)$ is a bijective function.

OR

Given that $A = \{1, 2, 3, \dots, 9\}$ $(a, b) R (c, d) \Leftrightarrow a + d = b + c$ for $(a, b) \in A \times A$ and $(c, d) \in A \times A$.

Let $(a, b) R (a, b)$

$$\Rightarrow a + b = b + a, \forall a, b \in A$$

Which is true for any $a, b \in A$

Hence, R is reflexive.

Let $(a, b) R (c, d)$

$$a + d = b + c$$

$$c + b = d + a \Rightarrow (c, d) R (a, b)$$

So, R is symmetric.

Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$

$$a + d = b + c \text{ and } c + f = d + e$$

$$a + d = b + c \text{ and } d + e = c + f \Rightarrow (a + d) - (d + e) = (b + c) - (c + f)$$

$$(a - e) = b - f$$

$$a + f = b + e$$

$$(a, b) R (e, f)$$

So, R is transitive.

Hence R is an equivalence relation.

Let $(a, b) R (2, 5)$, then

$$a + 5 = b + 2$$

$$a = b - 3$$

If $b < 3$, then a does not belong to A .

Therefore, possible values of b are > 3 .

For $b = 4, 5, 6, 7, 8, 9$

$$a = 1, 2, 3, 4, 5, 6$$

Therefore, equivalence class of $(2, 5)$ is

$$\{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}.$$

$$34. \text{ We have, } A = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix} \dots (i)$$

$$\therefore |A| = 1(-3) - 2(-2) + 0 = 1 \neq 0$$

$$\text{Now, } A_{11} = -3, A_{12} = 2, A_{13} = 2, A_{21} = -2, A_{22} = 1, A_{23} = 1, A_{31} = -4, A_{32} = 2 \text{ and } A_{33} = 3$$

$$\therefore \text{adj}(A) = \begin{vmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{vmatrix}^T = \begin{vmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{vmatrix}$$

$$\begin{aligned} \therefore A^{-1} &= \frac{\text{adj}A}{|A|} \\ &= \frac{1}{1} \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \\ \Rightarrow A^{-1} &= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \dots(i) \end{aligned}$$

Also, we have the system of linear equations as

$$x - 2y = 10$$

$$2x - y - z = 8$$

$$\text{and } -2y + z = 7$$

Now, the given system of equations can be rewritten in the form $AX=B$,

$$\text{where, } A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{and } B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

Since A is non singular, therefore given system of equations has a unique solution given by,

$$\begin{aligned} \therefore X &= A^{-1}B \\ \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} \\ &= \begin{bmatrix} -30 + 60 + 14 \\ -20 + 8 + 7 \\ -40 + 16 + 21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix} \\ \therefore x &= 0, y = -5 \text{ and } z = -3 \end{aligned}$$

35. Given Cartesian equations of lines

$$L_1 = \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$$

Line L_1 is passing through point (1, -1, 1) and has direction ratios (3, 2, 5)

Thus, vector equation of line L_1 is

$$\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$$

And

$$L_2 : \frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$$

Line L_2 is passing through point (2, -1, 1) and has direction ratios (2, 3, -2)

Thus, vector equation of line L_2 is

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} - 2\hat{k})$$

Now, to calculate distance between the lines,

$$\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} - 2\hat{k})$$

Here, we have

$$\vec{a}_1 = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{b}_1 = 3\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{b}_2 = 2\hat{i} + 3\hat{j} - 2\hat{k}$$

Thus,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 5 \\ 2 & 3 & -2 \end{vmatrix}$$

$$= \hat{i}(-4 - 15) - \hat{j}(-6 - 10) + \hat{k}(9 - 4)$$

$$\Rightarrow \vec{b}_1 \times \vec{b}_2 = -19\hat{i} + 16\hat{j} + 5\hat{k}$$

$$\Rightarrow |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-19)^2 + 16^2 + 5^2}$$

$$= \sqrt{361 + 256 + 25}$$

$$= \sqrt{642}$$

$$\vec{a}_2 - \vec{a}_1 = (2 - 1)\hat{i} + (1 + 1)\hat{j} + (-1 - 1)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = 1 + 2\hat{j} - 2\hat{k}$$

Now,

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (-19\hat{i} + 16\hat{j} + 5\hat{k}) \cdot (\hat{i} + 2\hat{j} - 2\hat{k})$$

$$= ((-19) \times 1) + (16 \times 2) + (5 \times (-2))$$

$$= -19 + 32 - 10$$

$$= 3$$

Thus, the shortest distance between the given lines is

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\Rightarrow d = \left| \frac{3}{\sqrt{642}} \right|$$

$$\therefore d = \frac{3}{\sqrt{642}} \text{ units}$$

As $d \neq 0$

Hence, given lines do not intersect each other.

OR

Here, it is given that the equation of lines

$$L_1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

$$L_2 = \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

Direction ratios of L_1 and L_2 are (3, -1, 1) and (-3, 2, 4) respectively.

Suppose general point on line L_1 is $P = (x_1, y_1, z_1)$

$$x_1 = 3s + 3, y_1 = -s + 8, z_1 = s + 3$$

and suppose general point on line L_2 is $Q = (x_2, y_2, z_2)$

$$x_2 = -3t - 3, y_2 = 2t - 7, z_2 = 4t + 6$$

$$\therefore \vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$= (-3t - 3 - 3s - 3)\hat{i} + (2t - 7 + s - 8)\hat{j} + (4t + 6 - s - 3)\hat{k}$$

$$\therefore \vec{PQ} = (-3t - 3s - 6)\hat{i} + (2t + s - 15)\hat{j} + (4t - s + 3)\hat{k}$$

Direction ratios of $\vec{PQ}$ are $((-3t - 3s - 6), (2t + s - 15), (4t - s + 3))$

PQ will be the shortest distance if it is perpendicular to both the given lines

Thus, by the condition of perpendicularity,

$$\Rightarrow 3(-3t - 3s - 6) - 1(2t + s - 15) + 1(4t - s + 3) = 0 \text{ and}$$

$$\Rightarrow -3(-3t - 3s - 6) + 2(2t + s - 15) + 4(4t - s + 3) = 0$$

$$\Rightarrow -9t - 9s - 18 - 2t - s + 15 + 4t - s + 3 = 0 \text{ and}$$

$$9t + 9s + 18 + 4t + 2s - 30 + 16t - 4s + 12 = 0$$

$$\Rightarrow -7t - 11s = 0 \text{ and}$$

$$29t + 7s = 0$$

Solving above two equations, we get,

$$t = 0 \text{ and } s = 0$$

therefore,

$$P = (3, 8, 3) \text{ and } Q = (-3, -7, 6)$$

Now distance between points P and Q is

$$d = \sqrt{(3 + 3)^2 + (8 + 7)^2 + (3 - 6)^2}$$

$$= \sqrt{(6)^2 + (15)^2 + (-3)^2}$$

$$= \sqrt{36 + 225 + 9}$$

$$= \sqrt{270}$$

$$= 3\sqrt{30}$$

Thus, the shortest distance between two given lines is

$$d = 3\sqrt{30} \text{ units}$$

Now, equation of line passing through points P and Q is,

$$\begin{aligned} \frac{x-x_1}{x_1-x_2} &= \frac{y-y_1}{y_1-y_2} = \frac{z-z_1}{z_1-z_2} \\ \therefore \frac{x-3}{3+3} &= \frac{y-8}{8+7} = \frac{z-3}{3-6} \\ \therefore \frac{x-3}{6} &= \frac{y-8}{15} = \frac{z-3}{-3} \\ \therefore \frac{x-3}{2} &= \frac{y-8}{5} = \frac{z-3}{-1} \end{aligned}$$

Thus, equation of line of shortest distance between two given lines is

$$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

Section E

36. i. $P(A) = \frac{1}{3}, P(A') = 1 - \frac{1}{3} = \frac{2}{3}$

$$P(B) = \frac{1}{2}, P(B') = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(\text{Both are selected}) = P(A \cap B) = P(A) \cdot P(B) = \frac{1}{3} \cdot \frac{1}{2}$$

$$P(\text{Both are selected}) = \frac{1}{6}$$

ii. $P(A) = \frac{1}{3}, P(A') = 1 - \frac{1}{3} = \frac{2}{3}$

$$P(B) = \frac{1}{2}, P(B') = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(\text{none of them selected}) = P(A' \cap B') = P(A') \cdot P(B') = \frac{2}{3} \cdot \frac{1}{2}$$

$$P(\text{Both are selected}) = \frac{1}{3}$$

iii. $P(A) = \frac{1}{3}, P(A') = 1 - \frac{1}{3} = \frac{2}{3}$

$$P(B) = \frac{1}{2}, P(B') = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(\text{none of them selected}) = P(A') \cdot P(B) + P(A) \cdot P(B') = \frac{2}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2}$$

$$P(\text{Both are selected}) = \frac{3}{6} = \frac{1}{2}$$

OR

$$P(A) = \frac{1}{3}, P(A') = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(B) = \frac{1}{2}, P(B') = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(\text{atleast one of them selected}) = 1 - P(\text{none selected}) = 1 - \frac{1}{3}$$

$$P(\text{atleast one of them selected}) = \frac{2}{3}$$

37. i. If ₹ 15000 is invested in bond X, then the amount invested in bond Y = ₹ (35000 - 15000) = ₹ 20000.

$$A = \text{Investment} \begin{bmatrix} X & Y \\ 15000 & 20000 \end{bmatrix}$$

$$\text{and } B = \begin{matrix} & \text{Interest rate} & \\ X & \begin{bmatrix} 10\% \\ 8\% \end{bmatrix} & \\ Y & \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix} & \end{matrix} = \begin{matrix} & \text{Interest rate} & \\ X & \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix} & \\ Y & \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix} & \end{matrix}$$

ii. The amount of interest received on each bond is given by

$$AB = \begin{bmatrix} 15000 & 20000 \end{bmatrix} \times \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$$

$$= [15000 \times 0.1 + 20000 \times 0.08] = [1500 + 1600] = 3100$$

iii. Let ₹ x be invested in bond X and then ₹ (35000 - x) will be invested in bond Y.

Now, total amount of interest is given by

$$\begin{bmatrix} x & 35000 - x \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix} = [0.1x + (35000 - x)0.08]$$

But, it is given that total amount of interest = ₹ 3200

$$\therefore 0.1x + 2800 - 0.08x = 3200$$

$$\Rightarrow 0.02x = 400 \Rightarrow x = 20000$$

Thus, ₹ 20000 invested in bond X and ₹ 35000 - ₹ 20000 = ₹ 15000 invested in bond Y.

OR

Let ₹ x invested in bond X, then we have

$$x \times \frac{10}{100} = 500 \Rightarrow x = 5000$$

Thus, amount invested in bond X is ₹ 5000 and so investment in bond Y be ₹ (35000 - 5000) = ₹ 30000

38. i. Since 'C' is cost of making tank

$$\therefore C = 70xy + 45 \times 2(2x + 2y)$$

$$\Rightarrow C = 70xy + 90(2x + 2y)$$

$$\Rightarrow C = 70xy + 180(x + y) \left[\because 2 \cdot x \cdot y = 8 \Rightarrow y = \frac{8}{2x} \Rightarrow y = \frac{4}{x} \right]$$

$$\Rightarrow C = 70x \times \frac{4}{x} + 180 \left(x + \frac{4}{x} \right)$$

$$\Rightarrow C = 280 + 180 \left(x + \frac{4}{x} \right)$$

ii. $x \cdot y = 4$

Volume of tank = length $\times$ breadth $\times$ height (Depth)

$$8 = x \cdot y \cdot 2$$

$$\Rightarrow 2xy = 8 \Rightarrow xy = 4$$

iii. For maximum or minimum

$$\frac{dC}{dx} = 0$$

$$\frac{d}{dx} \left(280 + 180 \left(x + \frac{4}{x} \right) \right) = 0 \Rightarrow 180 \left(1 + 4 \left(-\frac{1}{x^2} \right) \right) = 0$$

$$\Rightarrow 180 \left(1 - \frac{4}{x^2} \right) = 0 \Rightarrow 1 - \frac{4}{x^2} = 0$$

$$\Rightarrow \frac{4}{x^2} = 1 \Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$$\Rightarrow x = 2 \text{ (length can never be negative)}$$

OR

$$\text{Now, } \frac{d^2C}{dx^2} = 180 \left(+\frac{8}{x^3} \right)$$

$$\Rightarrow \left. \frac{d^2C}{dx^2} \right|_{x=2} = 180 \times \frac{8}{8} = 180 = +ve$$

Hence, to minimize C, $x = 2m$